An analytical model for a full wind turbine wake

Aidan Keane, Pablo E. Olmos Aguirre, Hannah Ferchland, Peter Clive, Daniel Gallacher

A: SgurrEnergy Ltd., 225 Bath Street, Glasgow G2 4GZ, Scotland

E: aidan.keane@sgurrenergy.com

Wake model

An analytical wind turbine wake model is proposed to predict the wind velocity distribution for all distances downwind of a wind turbine, including the near-wake. This wake model augments the Jensen model (Jensen 1983) and subsequent derivations thereof, and is a direct generalization of that recently proposed by Bastankhah and Porté-Agel (2014).

The model is derived by applying conservation of mass and momentum in the context of actuator disk theory, and assuming a distribution of the double-Gaussian type for the velocity deficit in the wake. The physical solutions are obtained by appropriate mixing of the waked- and freestream velocity deficit solutions, reflecting the fact that only a portion of the fluid particles passing through the rotor disk will interact with a blade.

The downwind wind speed is given by (Keane et al 2016)

$$U = U_\infty\left(1 - c\_C\_(x) f(r, \sigma(x))\right).$$

where

$$C\_(x) = \frac{M - \sqrt{M^2 - \frac{1}{2}NC_T d_0^2}}{2N}$$

and

$$M = 2\sigma^2 \exp(-\frac{1}{2}\tau^2) + \sqrt{2\pi}a\sigma[\mathrm{erfc}(\tau/\sqrt{2}) - 1]$$

$$N = \sigma^2 \exp(-\tau^2) + \frac{1}{2}\sqrt{\pi}a\sigma[\mathrm{erfc}(\tau) - 1]$$

$$\tau = r_0\sigma^{-1}.$$

The double-Gaussian profile is

$$f(r, \sigma(x)) = \frac{1}{2}[\exp D_+ + \exp D_-]$$

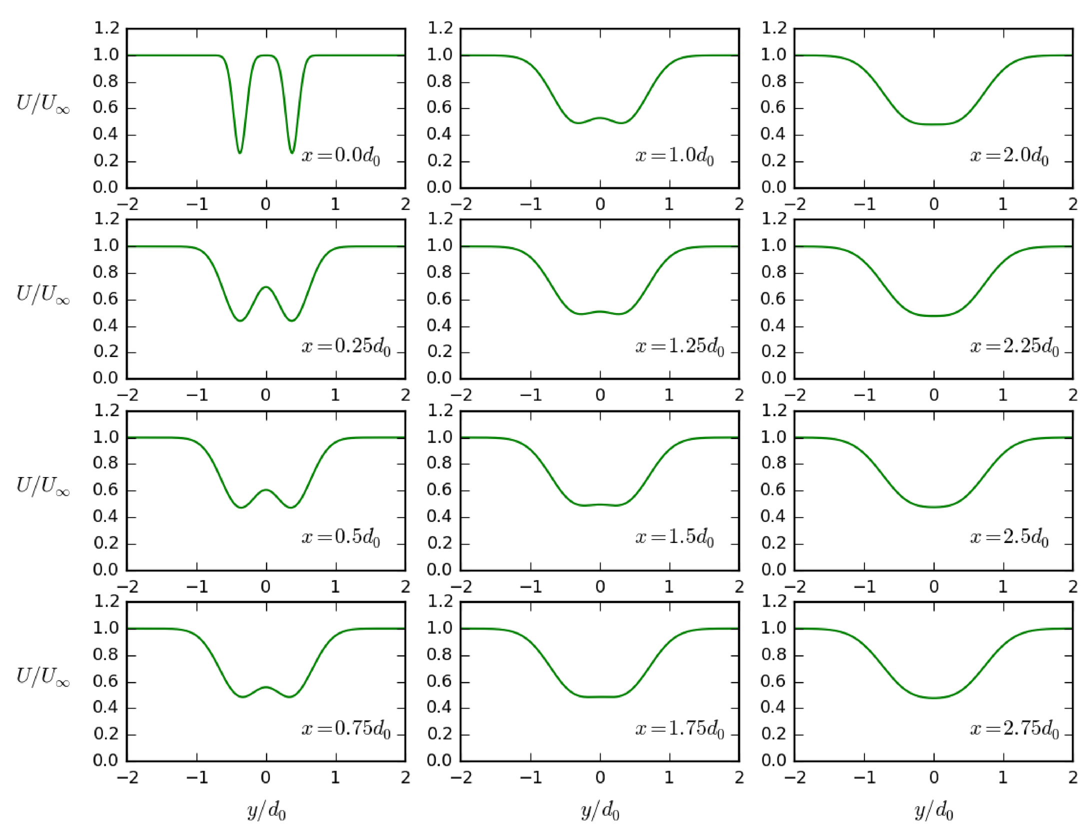
$$D_\pm = -\frac{1}{2}\sigma^{-2}(x)(r \pm r_0)^2.$$

The cross section is given by

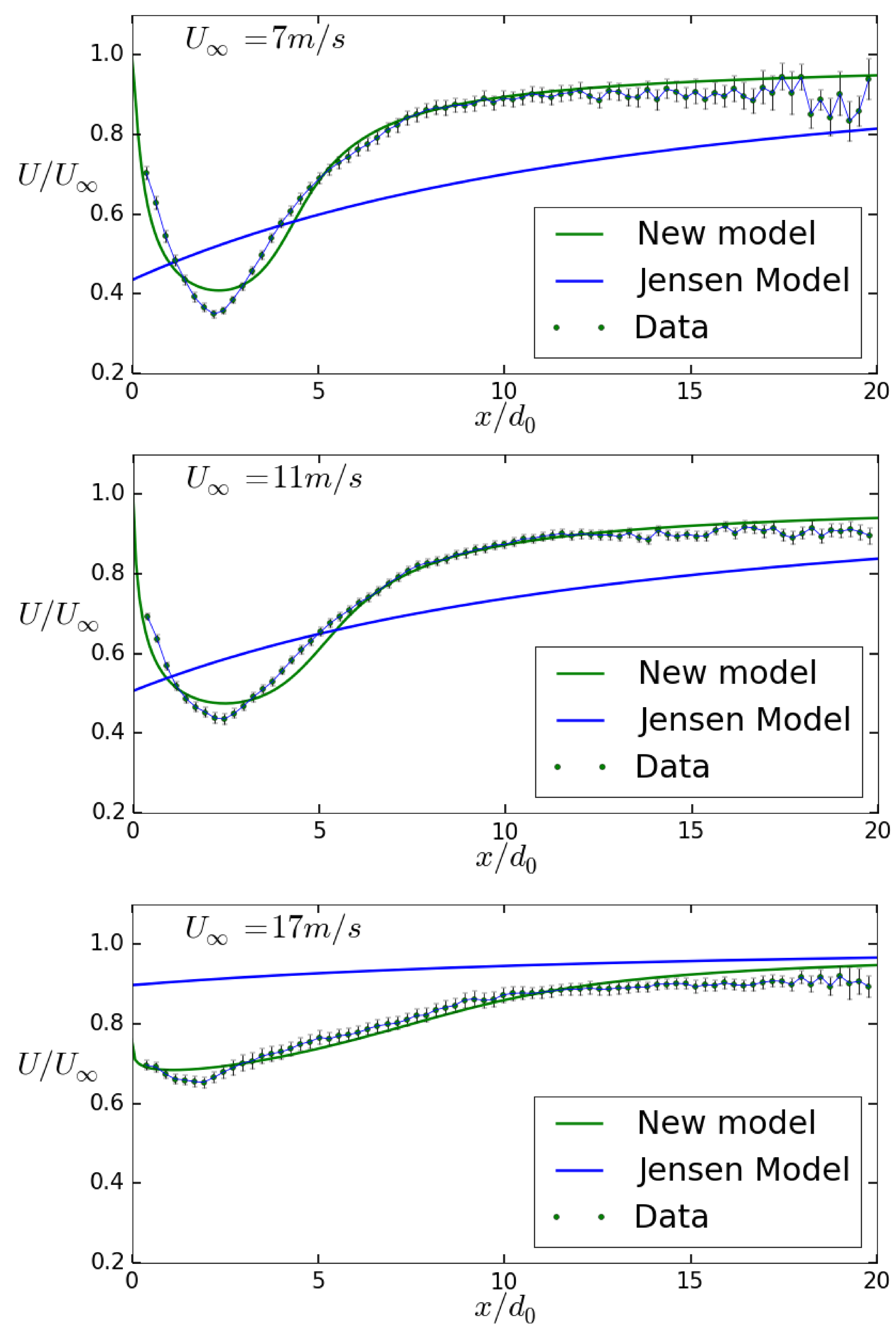
$$\sigma = k^* x^n + \epsilon$$

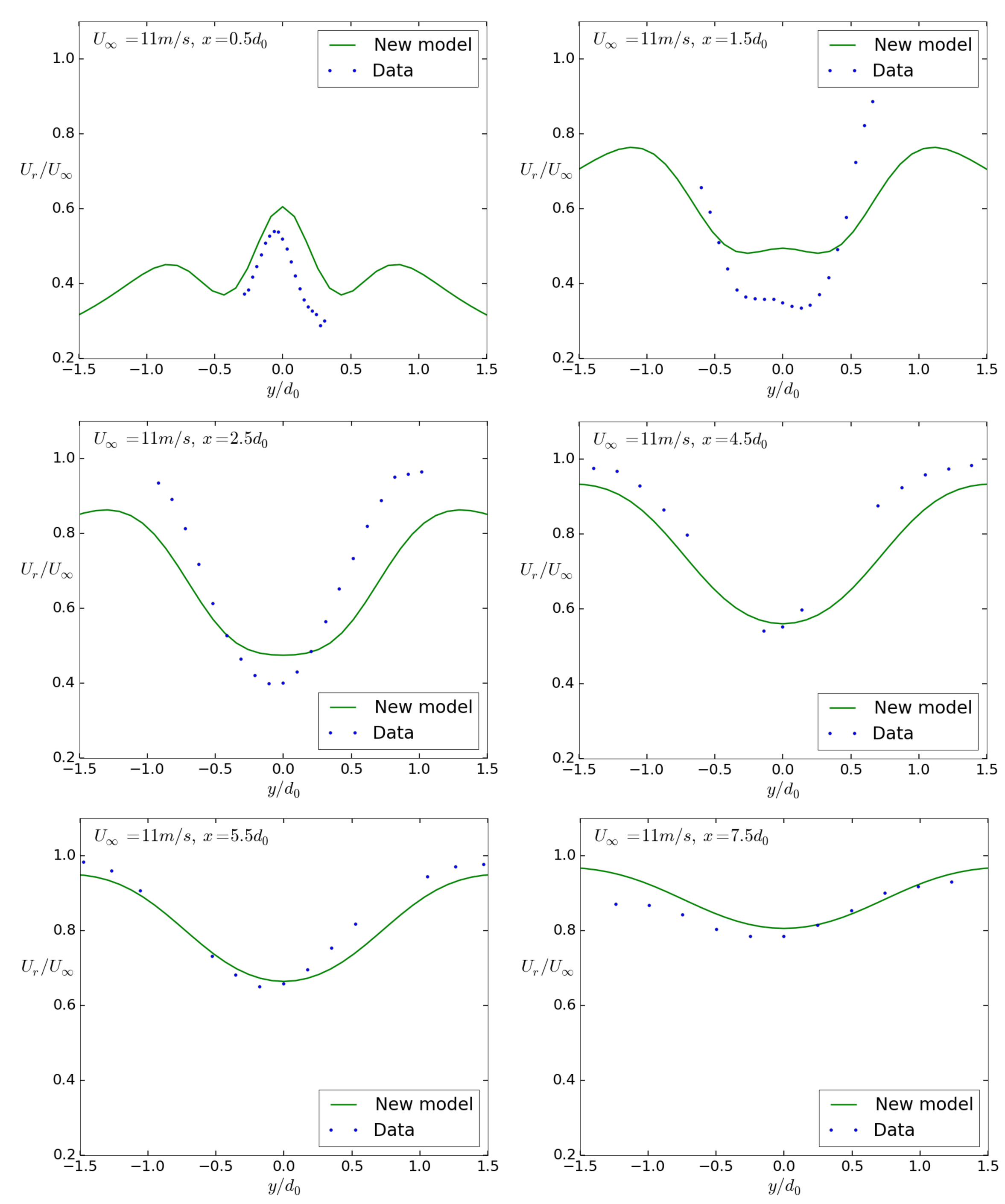
The real wake velocity solution depends upon several parameters: The wind turbine rotor diameter d_0, the radial location of the local minimum which has been determined empirically as $r_0 = 0.75\ d_0/2$; a, C_T are fixed by the wind turbine's thrust characteristics and vary with inflow wind speed U_∞, and the parameters k*, ε and c_ are obtained from fitting. The parameter values are given in Keane et al. 2016.

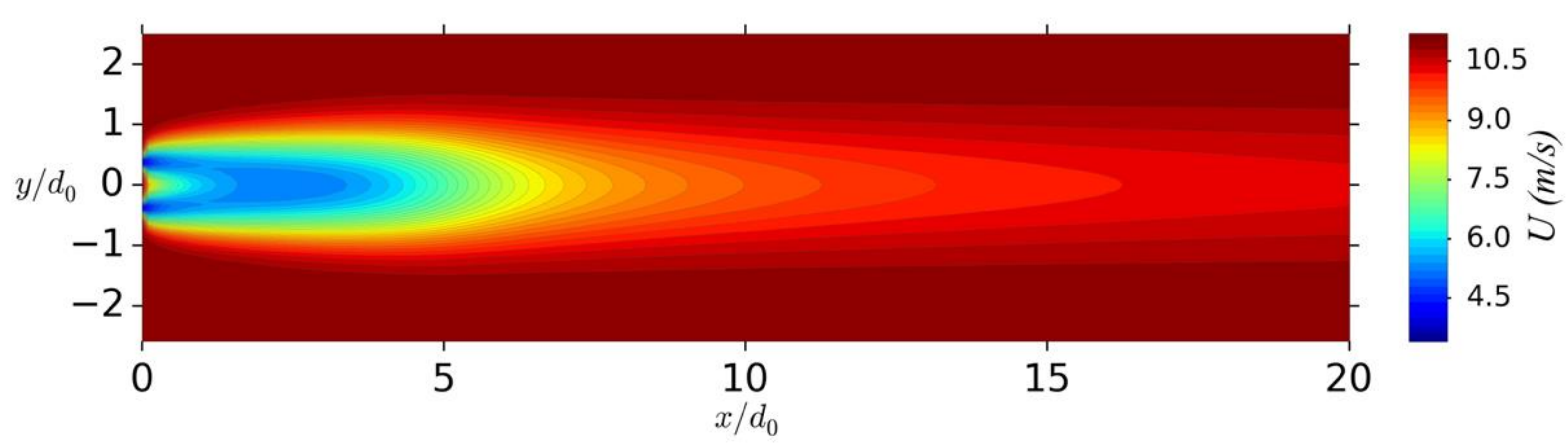
The lower figures show the wind velocity profiles for the hub height horizontal cross-sections through the wake model, for U_∞ = 11 m/s, for various downwind distances. There is a transition from double- to single-Gaussian distribution at a downwind distance of about 2.5 d_0.



Measurement versus theory







Overview

The figure on the left shows the horizontal normalized velocity profile for the wake centreline at hub height for the (10 minute averaged) wake data, the Jensen model (blue), and the newly proposed model (green).

The lower left figure shows the horizontal normalized nacelle-mounted LiDAR radial velocity profile for the wake cross-sections at hub height for the averaged wake data, and the newly proposed model, for U_∞ = 11 m/s and selected downwind distances x, based on the centreline hub height best fit parameter values given in Table 1 in Keane et al. (2016). The magnitude of the LiDAR measured radial velocity tends to zero for large distances from the wake centreline. This effect is due to the cosine factor arising as a result of the angular dependence of the LiDAR scan geometry.

This work was motivated by the desire to produce a model that more accurately predicts the near-wake region. The Jensen model provides a reasonable representation of the wake for the mid- and far-wake regimes, but there is a clear discrepancy in the near-wake, with the Jensen model predicting an unphysical drop-off in the centreline wake wind velocity. It is well known that the transverse velocity deficit profile can be represented by a single-Gaussian function for the mid- and far- wakes, but that in the near-wake the profile resembles a double-Gaussian function, with local minima at about 75% blade span. Thus, it is reasonable to consider a double-Gaussian function as a candidate for the transverse velocity deficit profile. Further, physically realistic solutions are obtained by subsequent, appropriate mixing of a wake velocity deficit solution and the freestream velocity. It is natural that such an adjustment should be required as only a fraction of the wind flow fluid particles passing through the rotor disk are affected by the blades. In summary, the wake model features:

- Double-Gaussian velocity deficit profile
- Mixing of waked- and freestream velocity solutions

The new proposed wake model is in close agreement with the measured data. The model performs reasonably well in the near wake region, exhibiting the expected local minima, and showing better agreement with increasing downwind distance. The model performs better than the corresponding Jensen model.

References

Jensen NO 1983 Risø National Laboratory Report M-2411

Bastankhah M and Porté-Agel F 2014 Renewable Energy 70 116

Aidan Keane et al 2016 J. Phys.: Conf. Ser. 753 032039

SUPERGEN Wind General Assembly – Durham – 27 April 2017