An analytical wind turbine wake model is proposed to predict the wind velocity distribution for all distances downwind of a wind turbine, including the near-wake. This wake model augments the Jensen model (Jensen 1983) and subsequent derivations thereof, and is a direct generalization of that recently proposed by Bastankhah and Porté-Agel (2014). The model is derived by applying conservation of mass and momentum in the context of actuator disk theory, and assuming a distribution of the double-Gaussian type for the velocity deficit in the wake. The physical solutions are obtained by appropriate mixing of the waked- and freestream velocity deficit solutions, reflecting the fact that only a portion of the fluid particles passing through the rotor disk will interact with a blade.

The downwind wind speed is given by (Keane et al 2016)

\[ U = U_\infty \left( 1 - c_c C_-(x) f(r, \sigma(x)) \right), \]

where

\[ C_-(x) = \frac{M - \sqrt{M^2 - 4NC_T d_0^2}}{2N} \]

and

\[ M = 2\sigma^2 \exp\left(-\frac{1}{2} \tau^2\right) + \sqrt{2\pi} \sigma \text{erfc}(\tau/\sqrt{2}) - 1 \]

\[ N = \sigma^2 \exp\left(-\frac{1}{2} \tau^2\right) + \frac{1}{2} \sqrt{2\pi} \sigma \text{erfc}(\tau) - 1 \]

\[ \tau = r_0 \sigma^{-1}. \]

The double-Gaussian profile is

\[ f(r, \sigma(x)) = \frac{1}{2} \exp D_+ + \exp D_- \]

\[ D_\pm = -\frac{1}{2} \sigma^{-2}(r \pm r_0)^2. \]

The cross section is given by

\[ \sigma = k^* x^m + \epsilon \]

The real wake velocity solution depends upon several parameters: The wind turbine rotor diameter \(d_0\), the radial location of the local minimum which has been determined empirically as \(r_0 = 0.75 \, d_0/2\); \(a, C_T\) are fixed by the wind turbine’s thrust characteristics and vary with inflow wind speed \(U_\infty\), and the parameters \(k^*, m, \epsilon\) and \(c_c\) are obtained from fitting. The parameter values are given in Keane et al. 2016.

The lower figures show the wind velocity profiles for the hub height horizontal cross-sections through the wake model, for \(U_\infty = 11\) m/s, for various downwind distances. There is a transition from double- to single-Gaussian distribution at a downwind distance of about 2.5 \(d_0\).