Numerical modelling of sub-sea turbine foundations

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SUPERGEN Wind Phase 2 - 3rd Training Seminar
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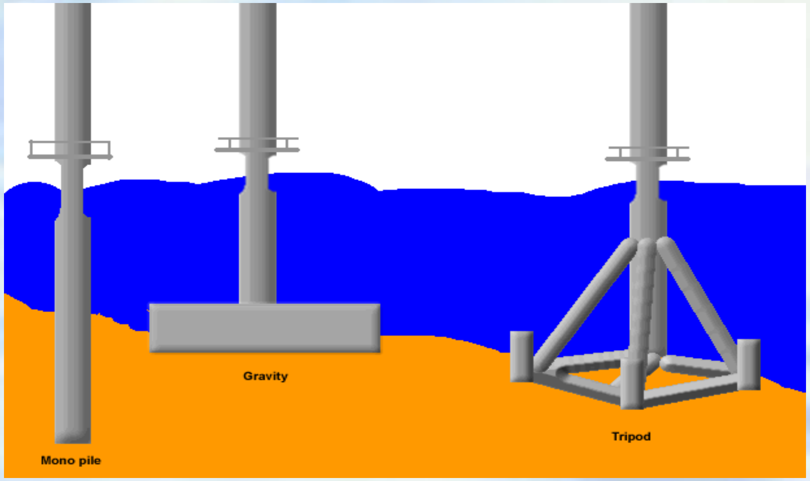
Outline

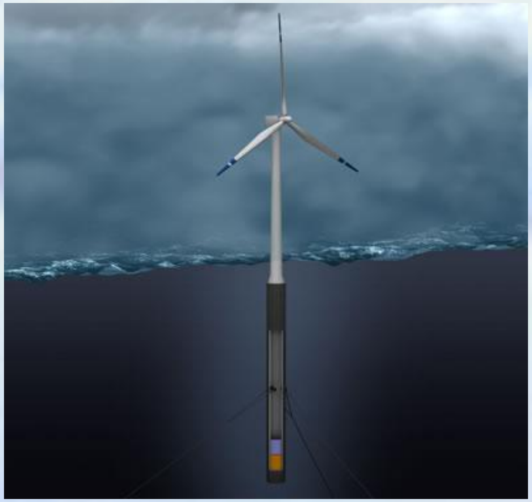
1. *Background*
2. *Flow solver*
3. *Sediment transport*
4. *Wave impact on structure*
5. *Summary*

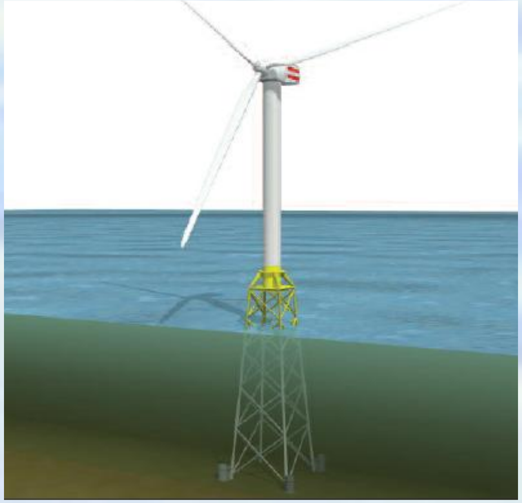
1. Background



Shallow water Intermediate water

Sources: the Danish Wind Industry Association



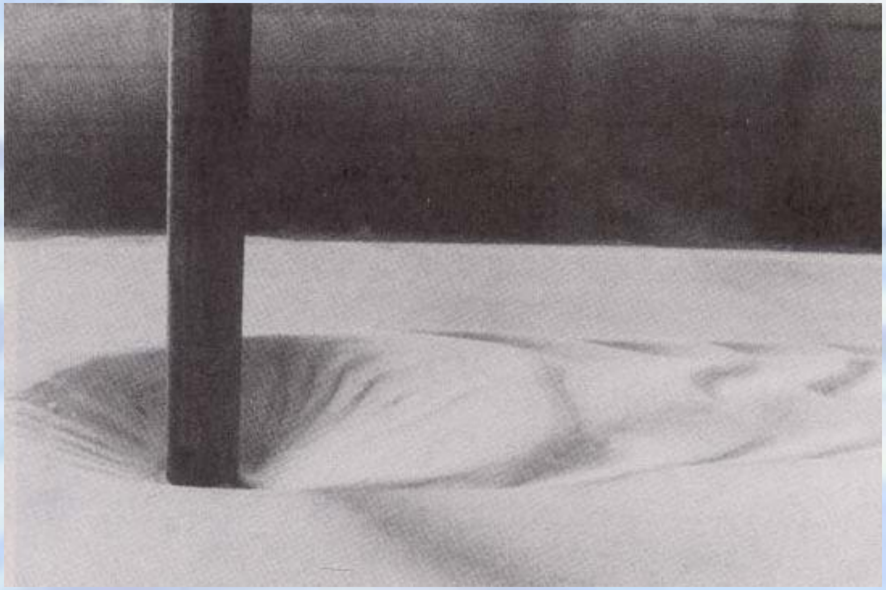


(Source: StatoilHydro)

Deep water: Monopile jacket type mount

Large offshore wind farms

1. Background



Flow pattern in the scour hole around a cylinder
(W.H.GRAF and I. ISTIARTO *JOURNAL OF HYDRAULIC RESEARCH*, VOL. 40, 2002)

Wave run-up on one of the towers of the Horns Rev wind farm
(Leen De Vos, et al. *Coastal Engineering*, *54, 2007*)

2. Flow solver

- Incompressible Navier-Stokes solver AMAZON-3D — Based on an artificial compressibility method.

- Surface-capturing method
  - Treats the free surface as a contact discontinuity in the density field, any special procedures to track the free surface are eliminated .

- Fully two phase approach which solves in **both** the air and the water fluid regions.

- Cartesian **c**ut **c**ell Method
  - flexibility for dealing with the complex geometries
  - no requirement to re-mesh globally and only requires changes locally at cells in the background Cartesian mesh that are cut by the moving boundary contour.

2. Flow solver

2.1 Governing equations

3D incompressible, Navier-Stoke equations with variable density:

$$\frac{\partial}{\partial t}\iiint_{\Omega}\mathbf{Q}\partial\Omega+\oint_{S}\mathbf{F}.\mathbf{n}\partial s=\iiint_{\Omega}\mathbf{B}\partial\Omega$$

where

$$\mathbf{Q}=\begin{bmatrix}\rho & \rho u & \rho v & \rho w & \dfrac{p}{\beta}\end{bmatrix}^T,\mathbf{F}=\mathbf{f}^I n_x+\mathbf{g}^I n_y+\mathbf{h}^I n_z,$$

$$\mathbf{B}=\begin{bmatrix}0 & 0 & 0 & -\rho g & 0\end{bmatrix}^T$$

$$\mathbf{f}^I=\begin{bmatrix}\rho u & \rho u^2+p & \rho uv & \rho uw & u\end{bmatrix}^T$$

$$\mathbf{g}^I=\begin{bmatrix}\rho v & \rho uv & \rho v^2+p & \rho vw & v\end{bmatrix}^T$$

$$\mathbf{h}^I=\begin{bmatrix}\rho w & \rho uw & \rho vw & \rho w^2+p & w\end{bmatrix}^T$$

β is the coefficient of artificial compressibility

2. Flow solver

2.2 Convective fluxes

- The convective flux ($\mathbf{F}_k$) is evaluated using Roe's approximate Riemann solver.

$$\mathbf{F}_k{}^{I} = \tfrac{1}{2}\left[\mathbf{F}^{I}\left(\mathbf{Q}_k^{+}\right) + \mathbf{F}^{I}\left(\mathbf{Q}_k^{-}\right) - \mathbf{R}\left|\wedge\right|\mathbf{L}\mathbf{Q}_k^{+} - \mathbf{Q}_k^{-}\right]$$

- To ensure second order accuracy, MUSCL reconstruction is used

$$\mathbf{Q}(x, y, z) = \mathbf{Q}_{i,j,k} + \Delta\mathbf{Q}_{i,j,k} \cdot \mathbf{r}$$

2.3 Time discretization

$$\left[I_m V + \frac{\partial R(\mathbf{Q}^{n+1,m})}{\partial \mathbf{Q}}\right]\left(\mathbf{Q}^{n+1,m+1} - \mathbf{Q}^{n+1,m}\right) = -\left[I_{ta}\frac{\left(\mathbf{Q}^{n+1,m} - \mathbf{Q}^{n,m}\right)V}{\Delta t} + R\left(\mathbf{Q}^{n+1,m}\right)\right]$$

where $I_m = diag\left[\tfrac{1}{\Delta\tau} + \tfrac{1}{\Delta t} \quad \tfrac{1}{\Delta\tau} + \tfrac{1}{\Delta t} \quad \tfrac{1}{\Delta\tau} + \tfrac{1}{\Delta t} \quad \tfrac{1}{\Delta\tau}\right]$

2. Flow solver

2.4 Turbulence equation

$$\frac{\partial \tilde{\nu}}{\partial t} + u_j \frac{\partial \tilde{\nu}}{\partial x_j} = \frac{1}{\sigma}\frac{\partial}{\partial x_j}\left[(\nu + \tilde{\nu})\frac{\partial \tilde{\nu}}{\partial x_j}\right] + C_{b1}\tilde{S}\tilde{\nu} + \frac{C_{b2}}{\sigma}\frac{\partial \tilde{\nu}}{\partial x_j}\frac{\partial \tilde{\nu}}{\partial x_j} - C_{w1} f_w \left(\frac{\tilde{\nu}}{d}\right)^2$$

DES length scale

d□:

$$\tilde{d} = \min(d, C_{DES}\Delta_{DES})$$

Where $\Delta_{DES} = \max(\Delta_x, \Delta_y, \Delta_z)$ and C_{DES}=0.65.

$C_{DES}\Delta_{DES} \ll d$ LES

$C_{DES}\Delta_{DES} > d$ RANS

Boundary conditions:

Inlet :

$$\frac{u(y)}{u_*} = \begin{cases} \dfrac{yu_*}{\nu} & \dfrac{yu_*}{\nu} \le 11.63 \\ \dfrac{1}{\kappa}\ln\left(9.0\dfrac{yu_*}{\nu}\right) & \dfrac{yu_*}{\nu} > 11.63 \end{cases} ; \; v(y) = 0$$

Outlet : $\partial / \partial n = 0$

Seabed and the cylinder surface: standard wall function

non-slip boundary

2. Flow solver

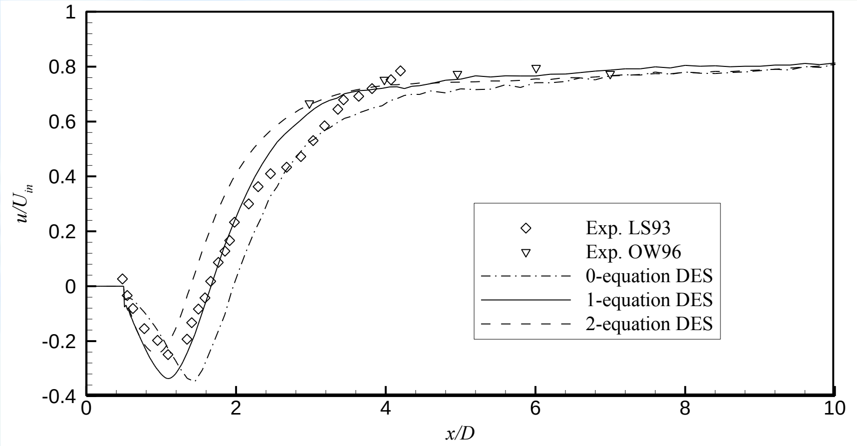
2.5 Numerical results of turbulent model:

Hydrodynamic parameters of flow over the circular cylinder at *Re*=3900 using different DES models

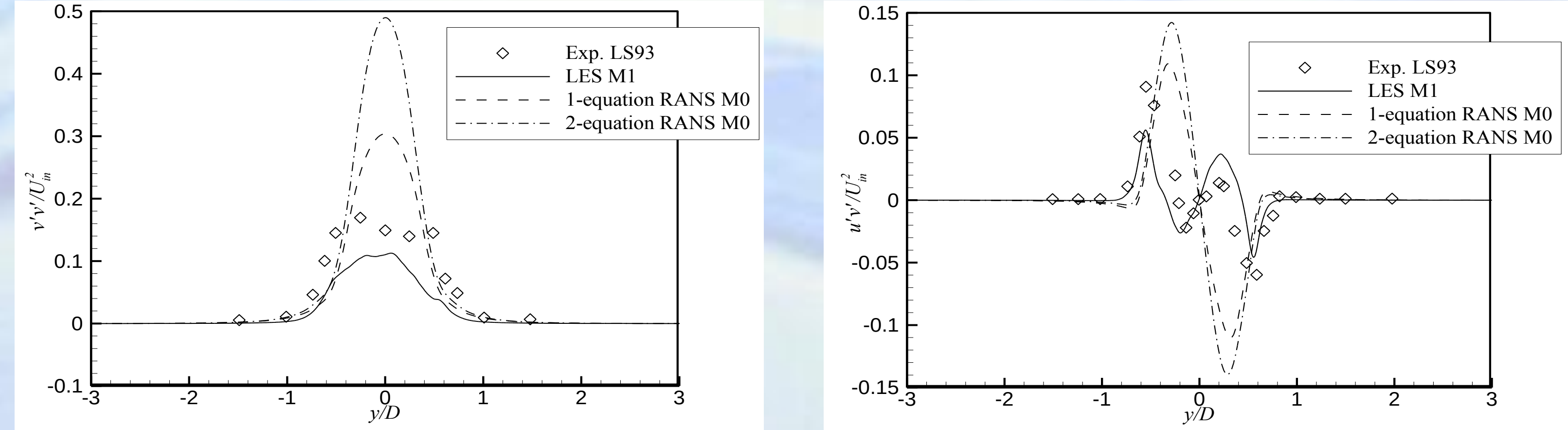
Case	C_D	C_{L_max}	C_{L_min}	*St*
Exp. Norberg 1987	0.98±0.05			0.215±0.005
0-equation_RANS	0.992	0.000	0.000	0.000
0-equation_DES	0.952	0.167	-0.199	0.229
1-equation_RANS	1.160	0.828	-0.836	0.234
1-equation_DES	1.046	0.424	-0.567	0.225
2-equation_RANS	1.067	0.757	-0.798	0.244
2-equation_DES	0.977	0.626	-0.660	0.234

Time history of drag force and lift force coefficients on the square cylinder

2.5 Numerical results of turbulent model:



Comparison of the mean centerline velocity component of the flow around the circular cylinder with different RANS based DES models



Comparison of the mean stress component at *x*=1.06m behind the circular cylinder with different RANS models: (a) spanwise Reynolds stress; (b) Reynolds shear stress

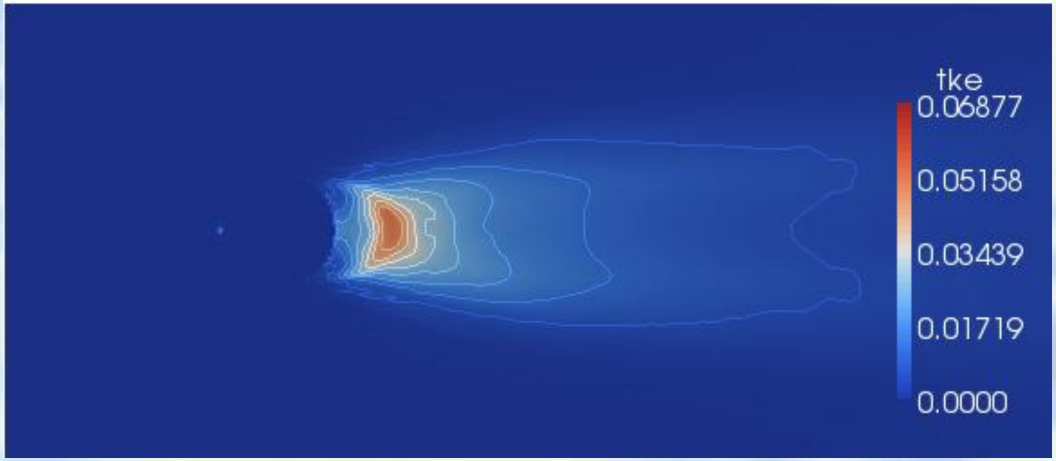
2. Flow solver

2.5 Numerical results of turbulent model:

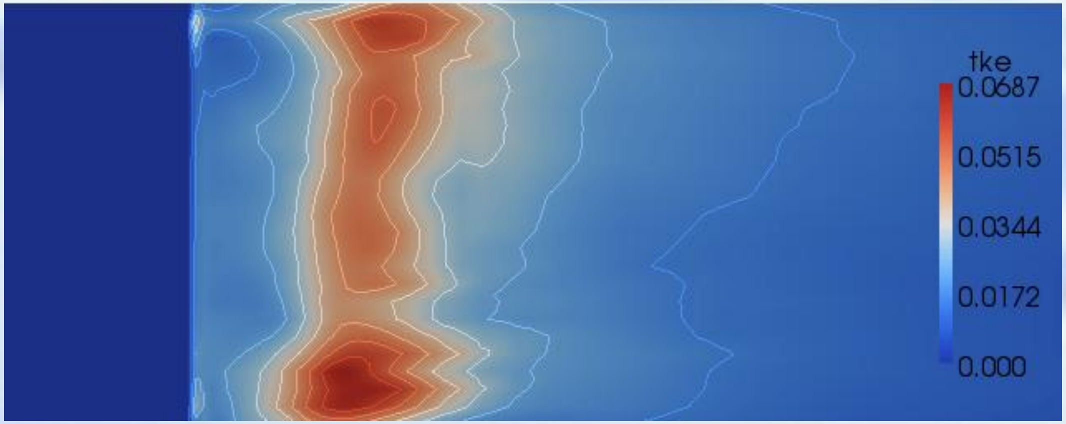
Velocity distribution at half water depth

2. Flow solver

2.5 Numerical results of turbulent model:



TKE distribution at z/h = 0.5 horizontal plane.



TKE distribution along the centerline

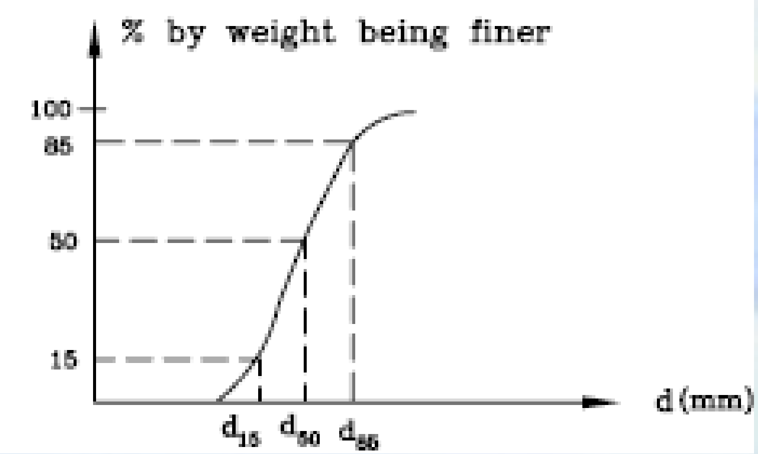
3. Sediment transport

3.1 Sediment properties

- *Density*

The density of natural sediments is $\rho_s = 2650\ kg/m^3$.
Therefore, the relative density is $s = \rho_s/\rho = 2.65$.

- *Grain size distribution*



Grain size distribution.

The median diameter of the sample is d_{50}, i.e. 50% of the grains by weight pass through.

D_* is the dimensionless grain size defined as

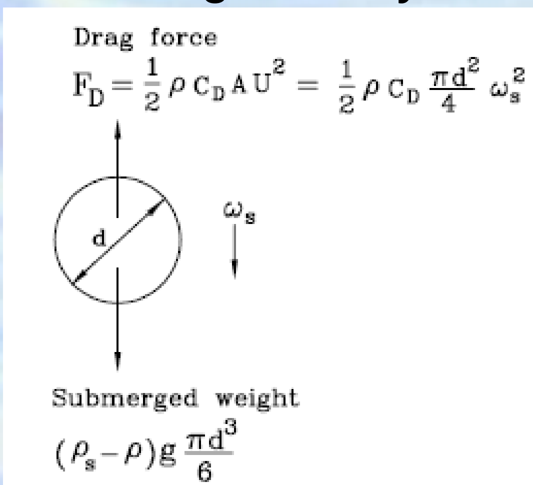
$$D_* = \left[\frac{g(s-1)}{\nu^2}\right]^{1/3} d_{50}$$

where ν is molecular kinetic viscosity of water.

3. Sediment transport

3.1 Sediment properties

- Settling velocity



Settling of a sphere in still water.

The force balance between the drag force and the submerged weight gives the settling velocity of the sphere:

$$\omega_s = \sqrt{\frac{4(s-1)gd}{3C_D}}$$

Laminar ($Re<0.5$) $C_D = 24/Re$ (by theory)
Turbulent ($Re>10^3$) $C_D \approx 0.4$ (by experiment)

Setting Velocity of Natural Sediments :

$$\omega_s = \frac{\upsilon}{d_{50}}\left[\sqrt{\frac{1}{4}\left(\frac{A}{B}\right)^{2/m} + \left(\frac{4D_*^3}{3B}\right)^{1/m}} - \frac{1}{2}\left(\frac{A}{B}\right)^{1/m}\right]^m$$

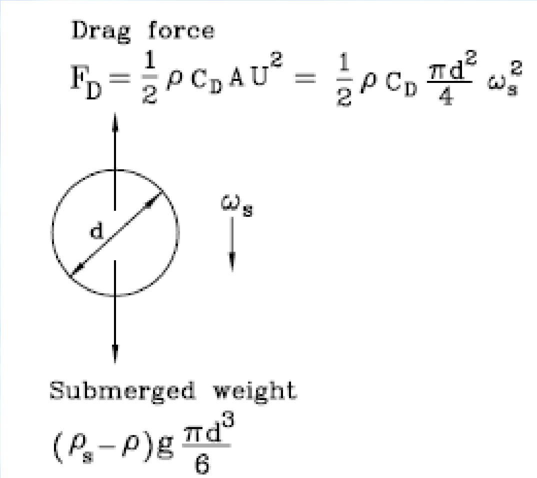
Calibration Coefficients
(depending on the authors)

Athhor(s)	Material	A	B	M
Dallavalle (1948)	Spherical particles	24.0	0.40	2.0
Julien (1995)	Natural sand	24.0	1.50	1.0
Soulsby (1997)	Natural sand	26.4	1.27	1.0
Cheng (1997)	Natural sand	32.0	1.00	1.5

3. Sediment transport

3.1 Sediment properties

- Settling velocity



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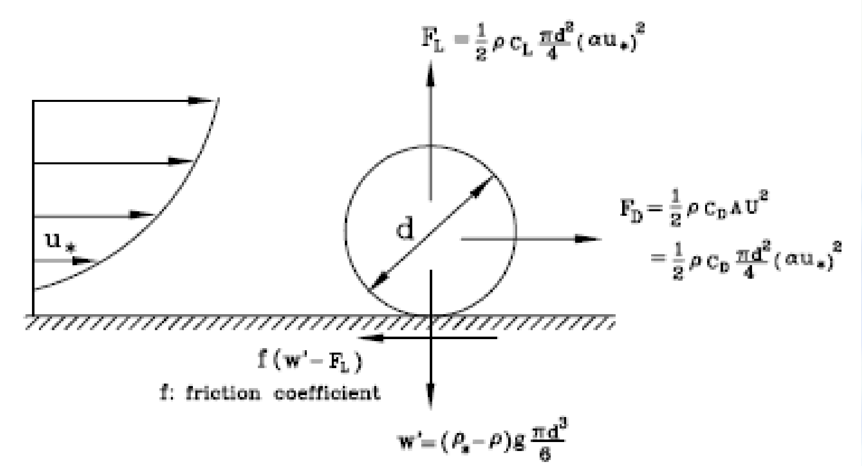
3. Sediment transport

3.1 Sediment properties

- Threshold of sediment



Forces acting on a grain resting on the bed.

with Shields parameter defined as

$$\theta = \frac{u_*^2}{(s-1)gd}$$ where u_* is friction velocity of flow close to the bed.

the sediment starts to move if

$u_* > u_{*,c}$ critical friction velocity $u_{*,c}$

or $\tau_b > \tau_{b,c}$ critical bottom shear stress $\tau_{b,c} = \rho\, u_{*,c}$

or $\theta > \theta_c$ critical bottom shear stress $\theta_c = \frac{u_{*,c}^2}{(s-1)gd}$

3. Sediment transport

3.2 Suspended sediment transport

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + (w - \omega_s)\frac{\partial c}{\partial z} = \frac{\partial}{\partial x}\left(\frac{\nu_t}{\sigma_c}\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_c}\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_c}\frac{\partial c}{\partial z}\right)$$

where *c* is the volumetric concentration of suspended sediment

u,v,w are velocity components

u_t is the eddy viscosity

s_c is the turbulent Schmidt number. (which is taken to be 0.8)

ω_s is the settling velocity of sediment.

$$\frac{\omega_s}{\omega_{s0}} = (1 - c)^m$$

m is a grain size related constant (*m* = 5)

ω_{s0} is the settling velocity of natural sands in clear water

we choose Soulsby's formulation (1997)

$$\omega_{S0} = \frac{\nu}{d_{50}}\left[\left(10.36^2 + 1.049 D_*^{\ 3}\right)^{0.5} - 10.36\right]$$

3. Sediment transport

3.2 Suspended sediment transport

Sediment bed boundary treatment:

a) Assume the sediment concentration follows Rouse profile

in the very close-to-wall region:

$$\frac{c}{c_a} = \left(\frac{\Delta_b}{H - \Delta_b} \frac{H - y}{y} \right) \qquad \text{with} \qquad Z = \frac{\sigma_c \omega_s}{\kappa u_*}$$

b) specify an effective boundary concentration (C_a):

$$c_a = 0.015 \frac{d_{50} T^{1.5}}{\Delta_b D_*^{0.3}}$$

Or

c) specify an upward sediment flux at boundary (E_a):

$$E_a = \omega_s c_a = 0.015 \omega_s \frac{d_{50} T^{1.5}}{\Delta_b D_*^{0.3}}$$

Δ_b is a reference level above the bed.

3. Sediment transport

3.3 Bed load transport

$$q_b = \begin{cases} 0.053\left[(s-1)g\right]^{0.5} \dfrac{d_{50}^{\ 1.5} T^{2.1}}{D_*^{0.3}} & T > 0 \\ 0 & T \leq 0 \end{cases}$$

where q_b is the volumetric bed load transport rate per unit width.

T is the non-dimensional excess shear stress or transport stage parameter.

$$T = \frac{u_{*s}^2 - u_{*cr}^2}{u_{*cr}^2}$$

where u_{*s} is the friction velocity component due to skin friction

u_{*cr} is the threshold bed friction velocity for motion of sediment.

An algebraic expression of the Shields diagram is used to calculate u_{*cr}, (Soulsby and Whithouse (1997))

$$\theta_{cr0} = \frac{0.3}{1 + 1.2D_*} + 0.055\left[1 - \exp\left(-0.02D_*\right)\right]$$

where θ_{cr0} is the threshold Shields parameter on horizontal bed.

3. Sedimenttransport

3.3 Bed load transport

For a sloping bed, the critical threshold Shields parameter θ_{cr0} is modified as: (*Zhao and Teng, 2001; Roulund et al,. 2005*)

$$\theta_{cr} = \theta_{cr0}\left[\sqrt{\cos^2\beta - \frac{\sin^2\beta\sin^2\alpha}{\mu_s^2}} - \frac{\cos\alpha\cos\beta}{\mu_s}\right]$$

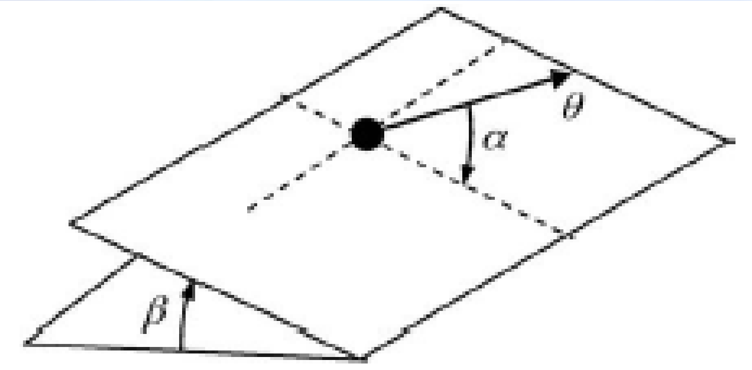
In which θ_{cr} is the critical threshold Shields parameter for flat bed.

$\mu_s = \tan\varphi_s$ is the static friction coefficient.

φ_s is the angle of repose of sediment.

β is the bed slope angle

α is the angle between the down slope direction and the bed shear tress.



Critical Shields parameter on a slope

3. Sediment transport

3.4 Bed deformation

$$\frac{\partial z_b}{\partial t} = \frac{1}{1 - p_0}\left[-\frac{\partial q_{tx}}{\partial x} - \frac{\partial q_{ty}}{\partial y} - \frac{\partial}{\partial t}\left(\int_{z=z_b}^{z_b+H} c dz \right) \right]$$

where p_0 is the porosity of the bed material.

z_b is the seabed elevation

q_{tx} and q_{ty} are the total sediment transport rates in x and y directions

Sand slide:

adjust the local bed slope when it exceed the angle of repose of the sediment in the simulation.

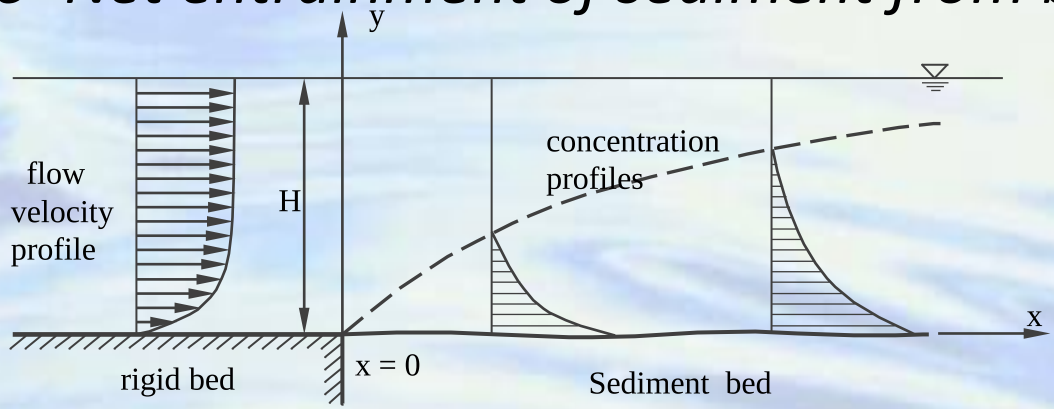
3. Sediment transport

3.5 Sequence of simulation

1) Start from an initial channel bed and flow field;
2) Solving the turbulent model to get the velocity, pressure and turbulent quantities;
3) Compute the suspended sediment concentration;
4) Compute the bed load and suspended sediment transport rates;
5) Calculate the bed deformation and the bed elevation and adjust the grid and the geometry parameters;
6) Return to step 2) and repeat the preceding calculation until a specified time has been reached.

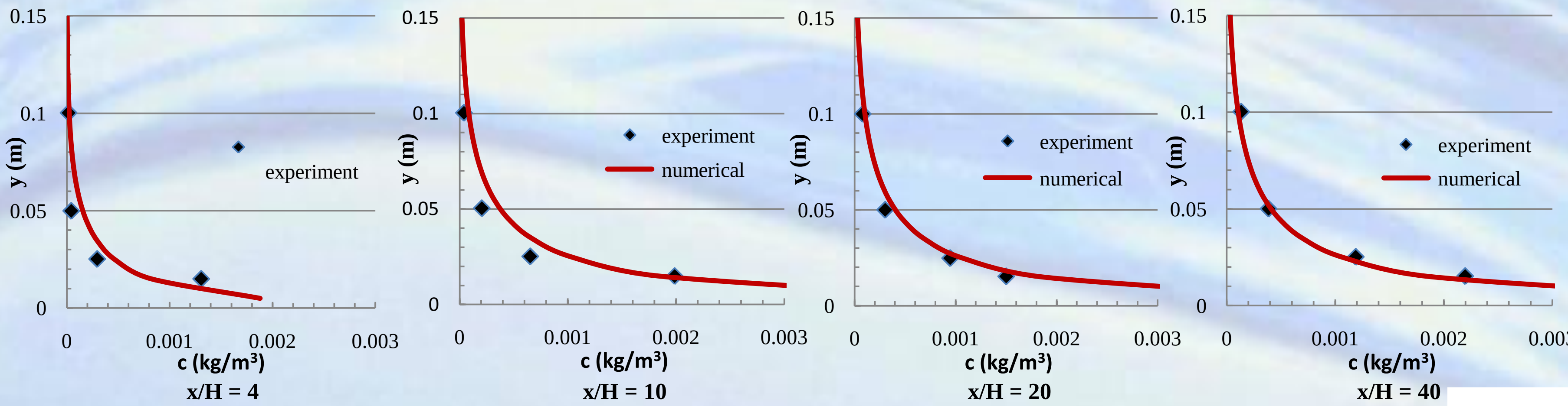
3. Sediment transport

3.6 Net entrainment of sediment from bed:



$H = 0.25$ m
$U = 0.67$ m/s
$\omega_s = 0.022$ m/s
$k_s = 0.01$m

Schematic definition of the problem

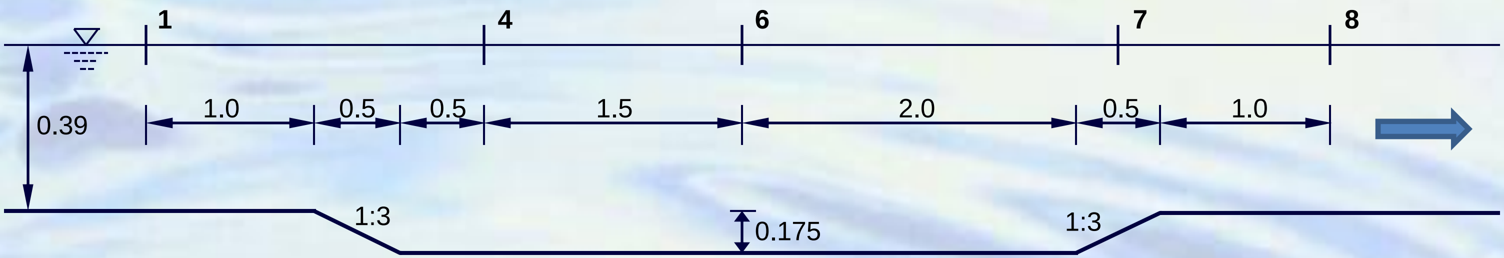


Comparison of numerical and the experiment suspended sediment concentrations

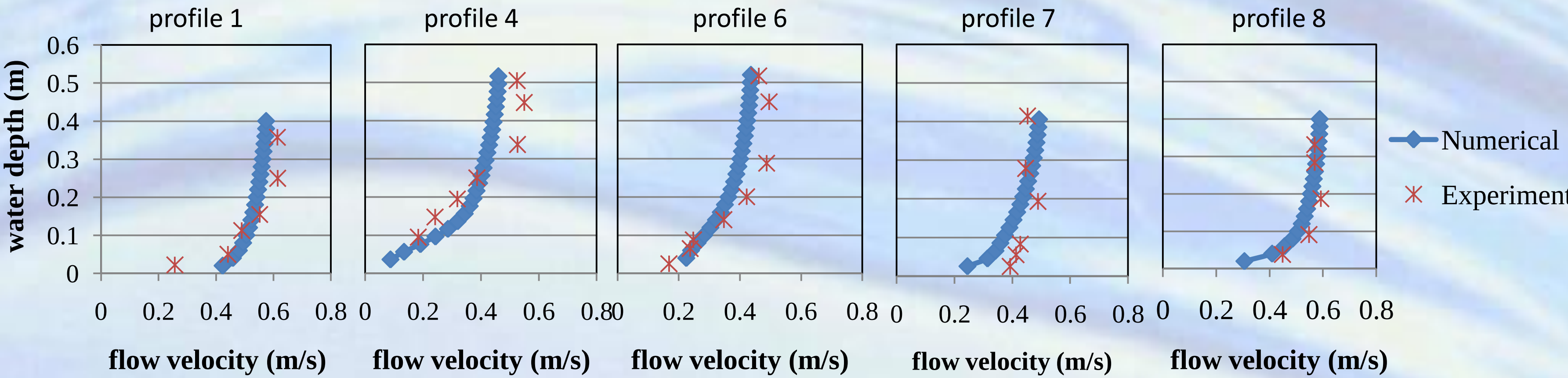
3. Sediment transport

3.6 Sediment transport in a trench:

One of Rijn experimental test (1985, flow past a trench)



Schematic trench and measurement stations

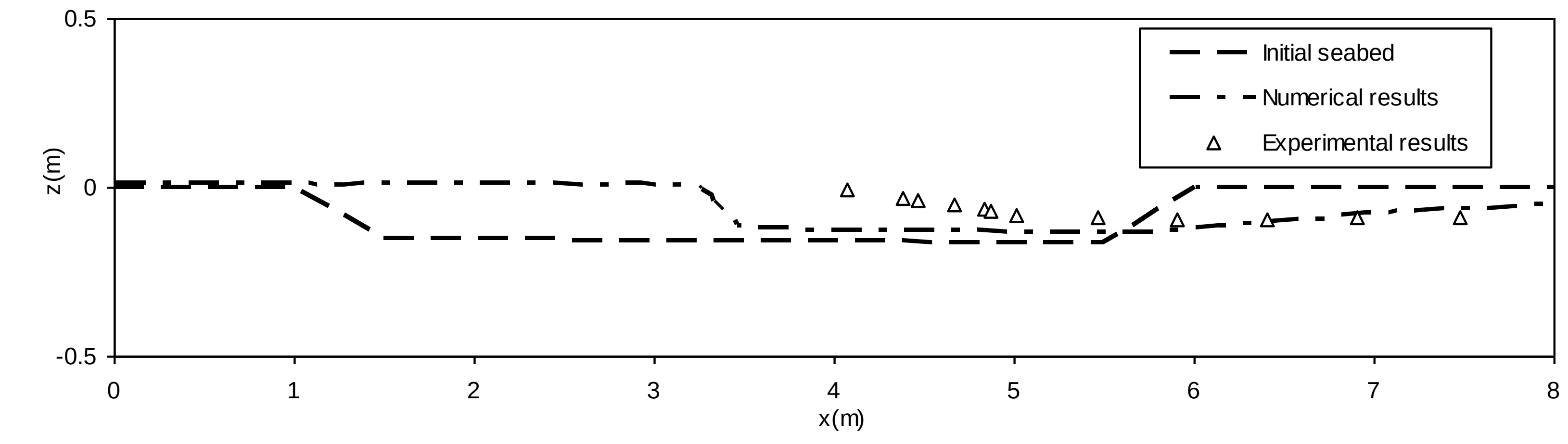


Velocity distribution at different profiles

3. Sediment transport

3.6 Sediment transport in a trench:

Sediment concentration developing in first 90s



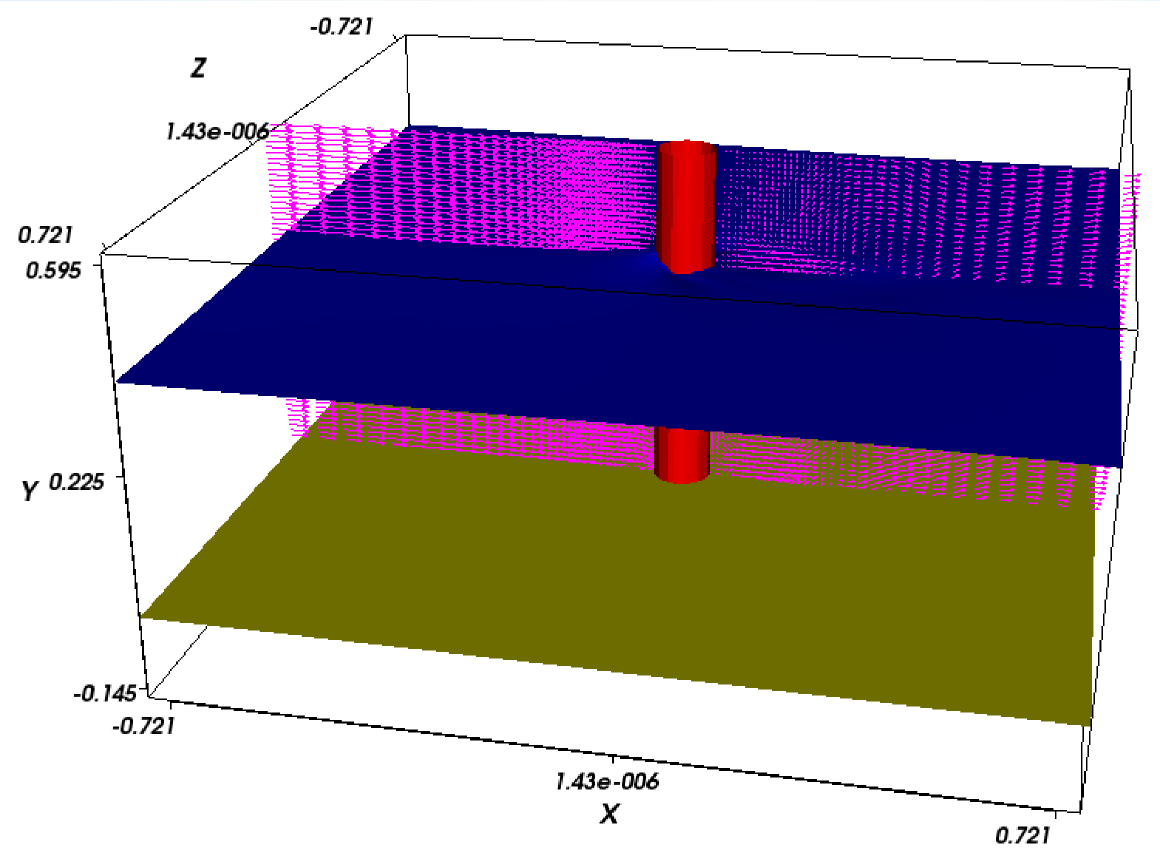
Comparison of morphological change after 15 hours

3. Sediment transport

3.7 Scour simulation around a circular cylinder:

Input velocity distribution is using logarithmic flow velocity profile:

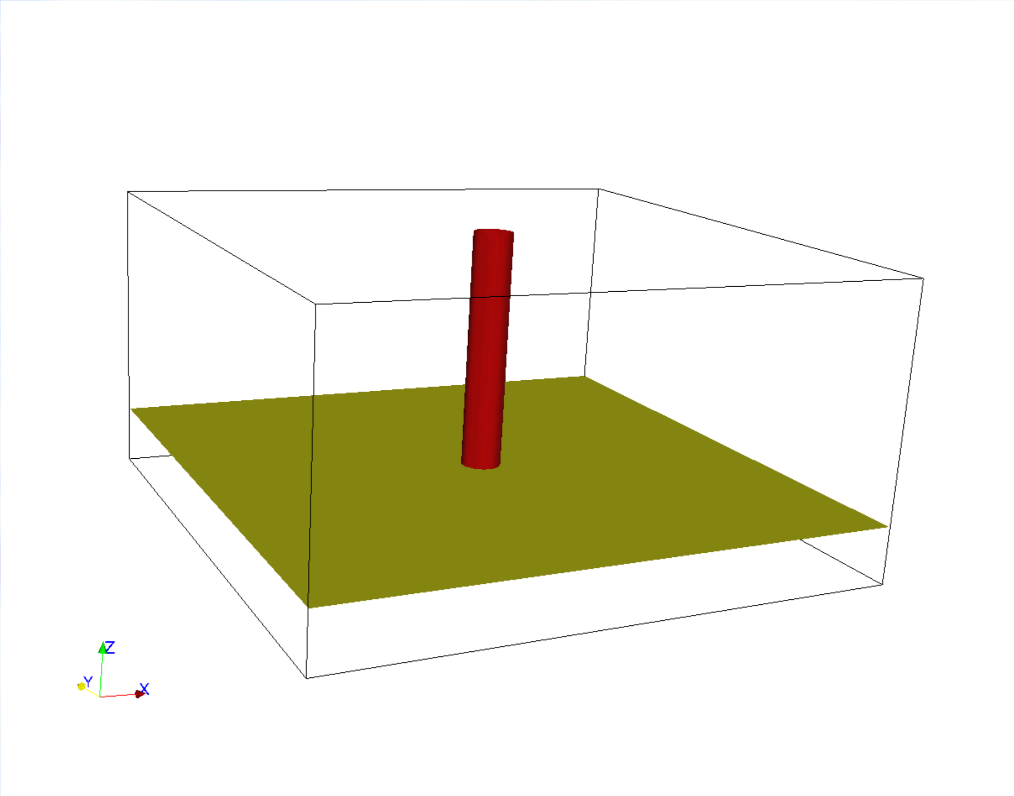
$$u_z = \frac{u_*}{\kappa} \ln \frac{z}{z_0}$$



Computational set up for scour simulation

3. Sediment transport

3.7 Scour simulation around a circular cylinder:



Scour hole development at t = 2400s model scale

Total number of cells:
80 x 80 x 75
Computational time for 2400 seconds simulation: 25 days

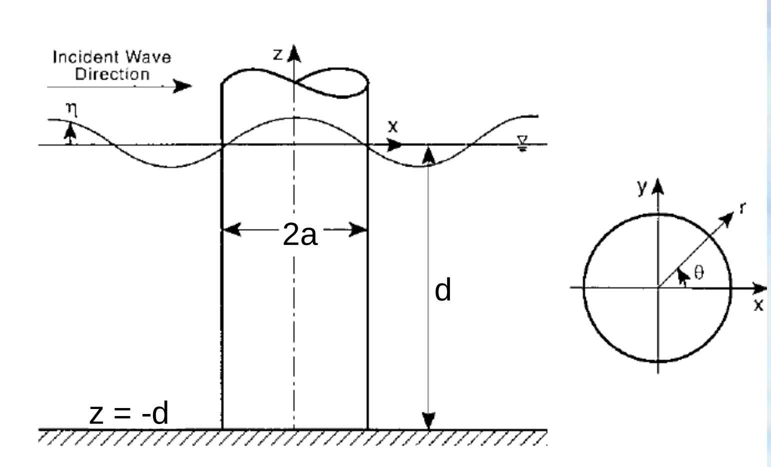
4. Wave impact on structure

4.1 Wave force calculation:

- Wave diffraction theory:

The first order wave force is:

$$\bar{F} = \frac{F}{F_0} = \frac{2}{ka\left[J_1'^2(ka) + Y_1'^2(ka)\right]^{1/2}} \frac{e^{-i\omega t}}{2} + c.c.$$



Definition sketch of wave forces on a vertical circular cylinder.

The second-order force are:

a) steady quadratic force components

$$F_s = \frac{\rho g a H^2}{2\pi (ka)^2}\left(1 + \frac{2kd}{\sinh 2kd}\right)\sum_{n=1}^{\infty}\left[\frac{n(n-1)}{(ka)^2} - 1\right] A_n A_{n-1} \sin(\varphi_n - \varphi_{n-1})$$

b) Oscillatory quadratic force components

$$F_o = \frac{\rho g a H^2}{2\pi (ka)^2}\left[\left(1 + \frac{2kd}{\sinh 2kd}\right)\sum_{n=1}^{\infty}\frac{n(n-1)}{(ka)^2} A_n A_{n-1} e^{i\left[(2n-1)\frac{\pi}{2} - \varphi_n - \varphi_{n-1}\right]}\right.$$

$$\left. + \left(3 - \frac{2kd}{\sinh 2kd}\right)\sum_{n=1}^{\infty}\frac{n(n-1)}{(ka)^2} A_n A_{n-1} e^{i\left[(2n-1)\frac{\pi}{2} - \varphi_n - \varphi_{n-1}\right]}\right]\frac{e^{-2i\omega t}}{2} + c.c$$

- Numerical calculation:

$$F(t) = -\int_{-d}^{\eta}\int_{-\pi}^{\pi} P(a, \theta, z, t)\, a \cos\theta \, d\theta \, dz$$

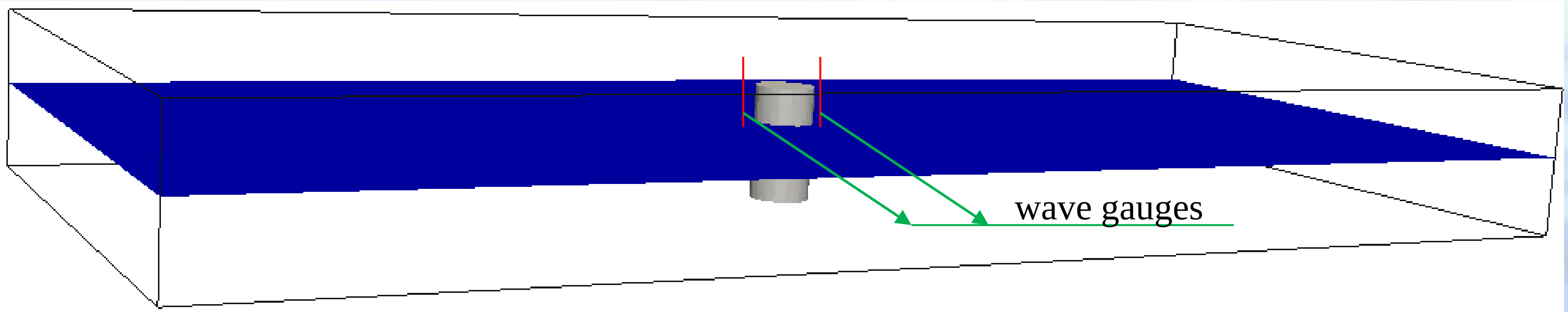
4. Wave impact on structure

4.2 validation test:

The NWT outer dimensions: $8 \times 3.6 \times 0.9\ m^3$

water depth : h = 0.45m

diameter of the cylinder : d = 0.325m

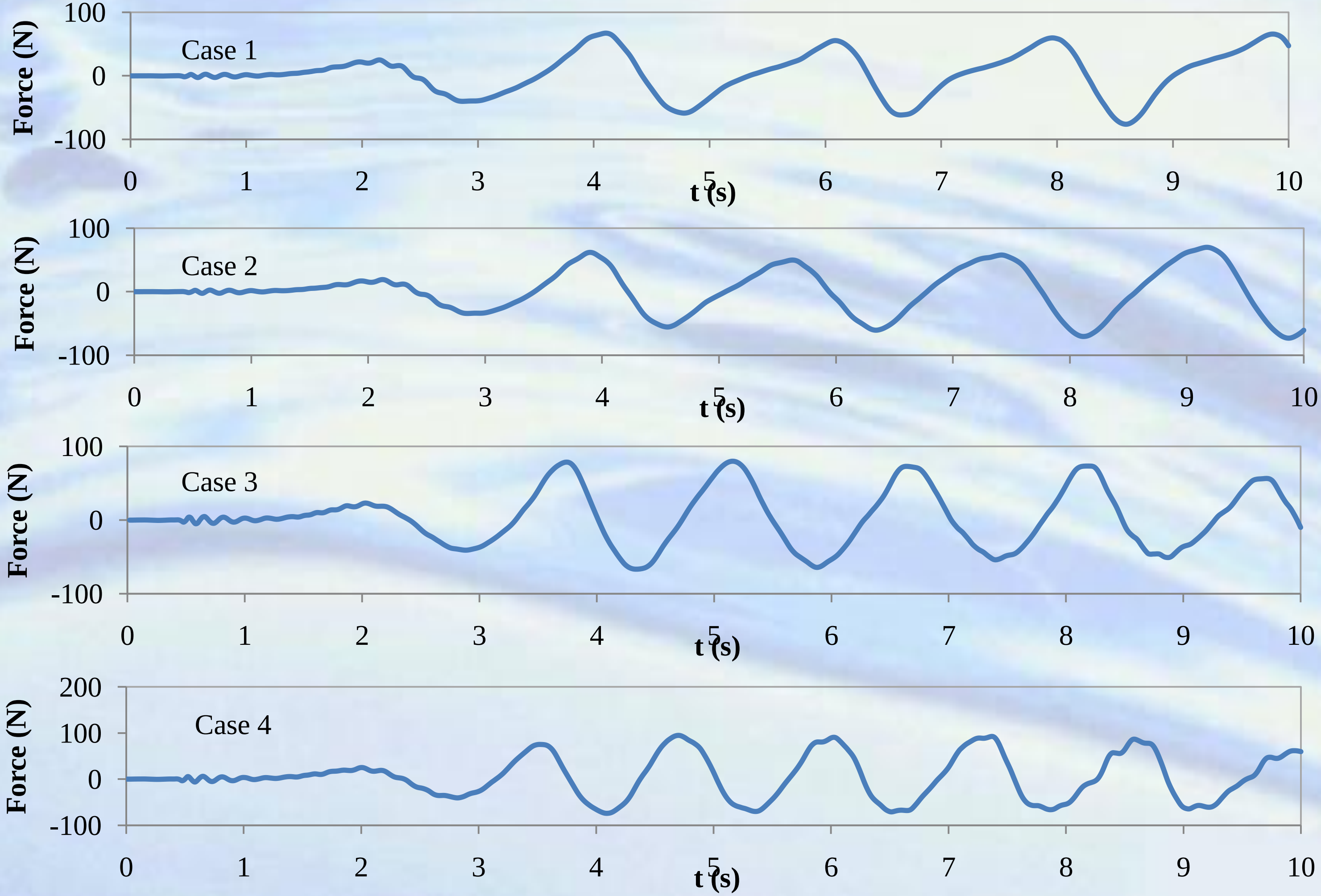


	Case1	Case2	Case3	Case4
Wave amplitude (m)	0.0535	0.048	0.0621	0.074
Wave period (s)	1.95	1.75	1.50	1.25
Scattering parameter ka	0.271	0.308	0.374	0.481

Experiments and Theoretical analyses conducted by D.L. Kriebel (1998) and J.R. Chaplin *et al*. (1997)

4. Wave impact on structure

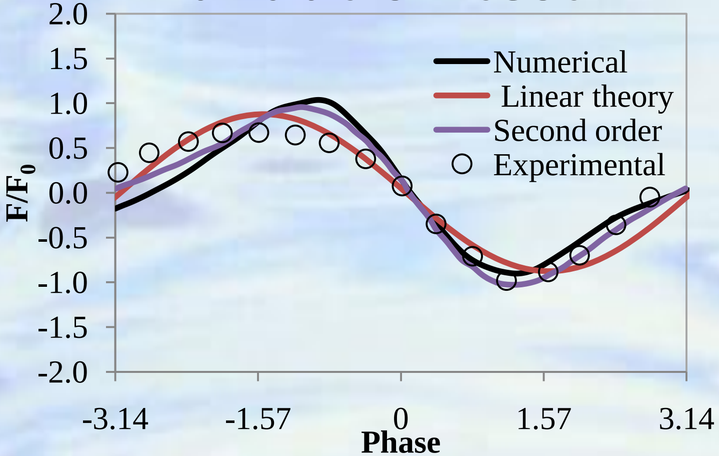
4.2 validation test:



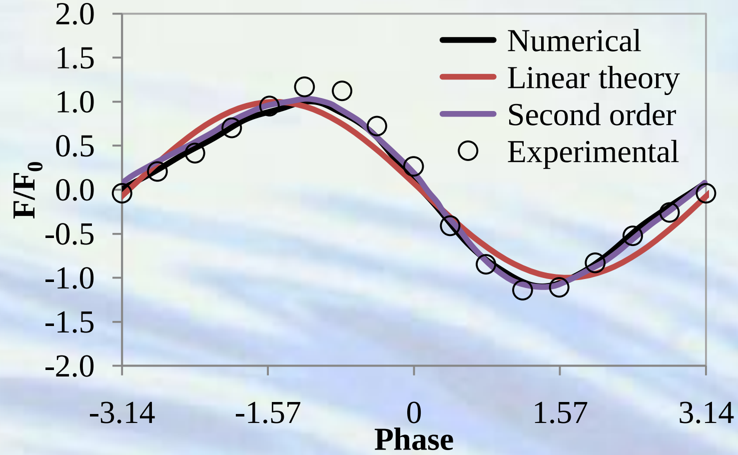
Time history of horizontal force on cylinder

4. *Wave impact on structure*

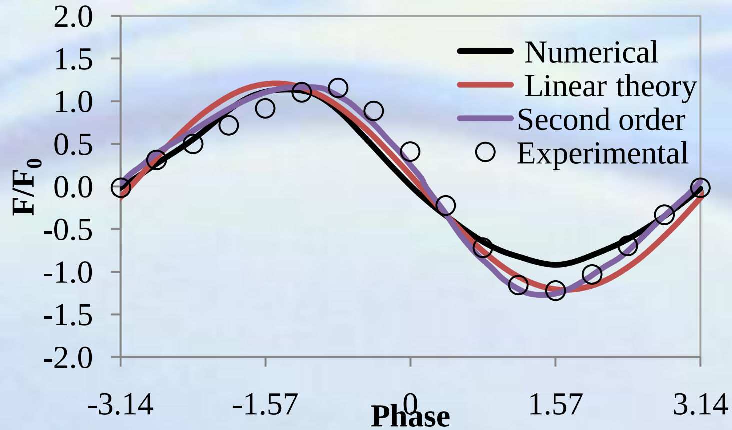
4.2 validation test:



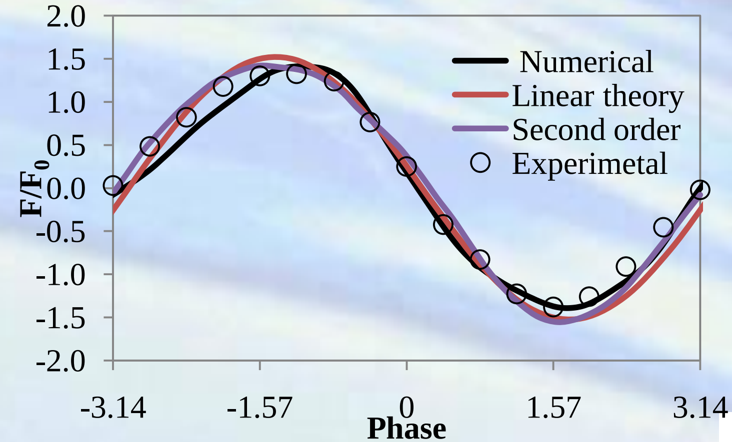
(a) case1: ka = 0.271, kH = 0.178



(b) case2: ka = 0.308, kH = 0.182



(c) case3: ka = 0.374, kH = 0.286

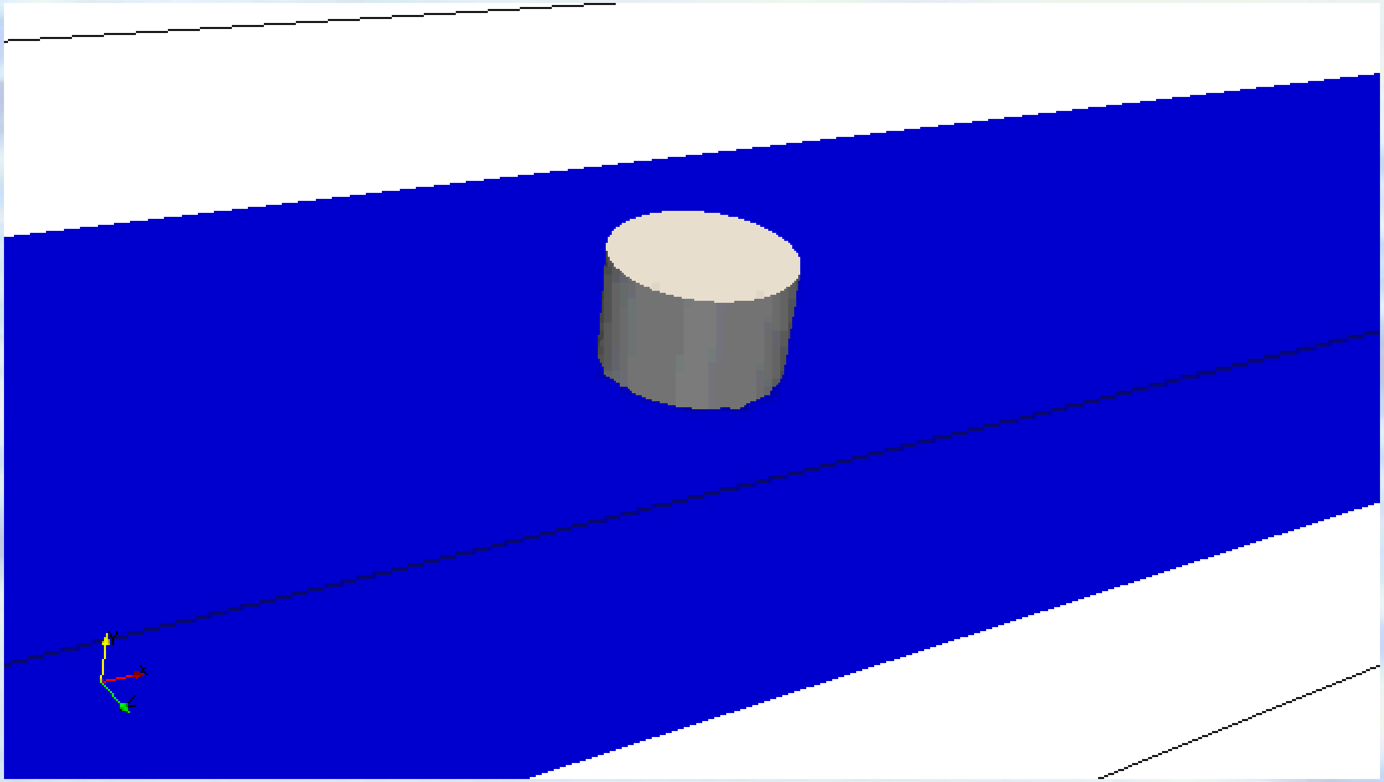


(d) case4: ka = 0.481, kH = 0.438

Comparison of wave force time series for various combinations of ka and kH with d/a=2.77

4. Wave impact on structure

4.2 validation test:

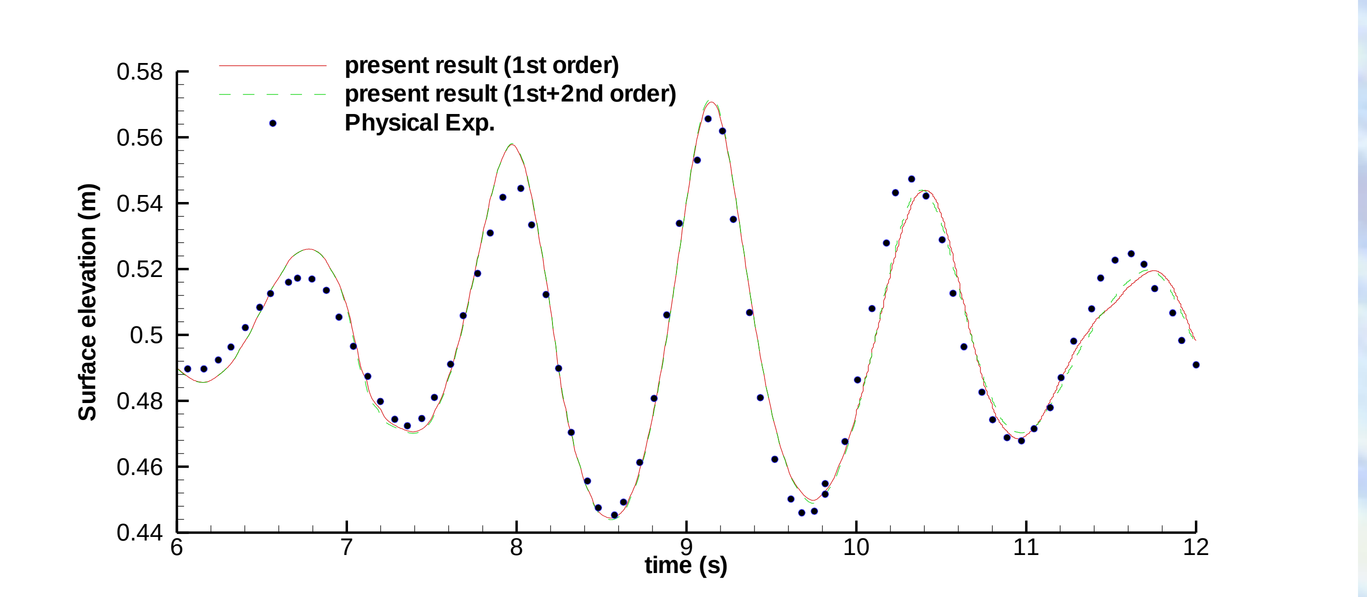


Wave surface around the vertical cylinder for case1

4. Wave impact on structure

4.3 Extreme wave generating test

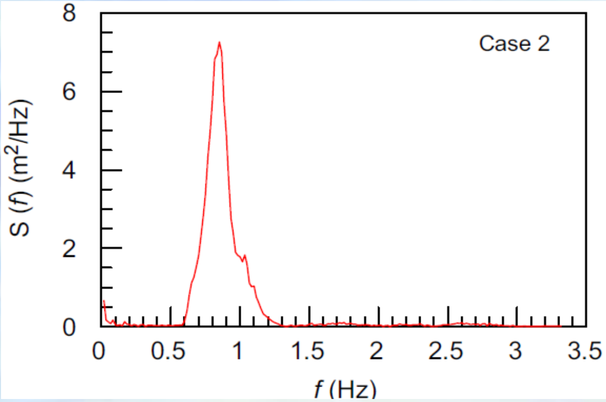
- Wave maker used by J.F. Dalzell (1999)
- Experiments conducted by Ning *et al*. (2008)



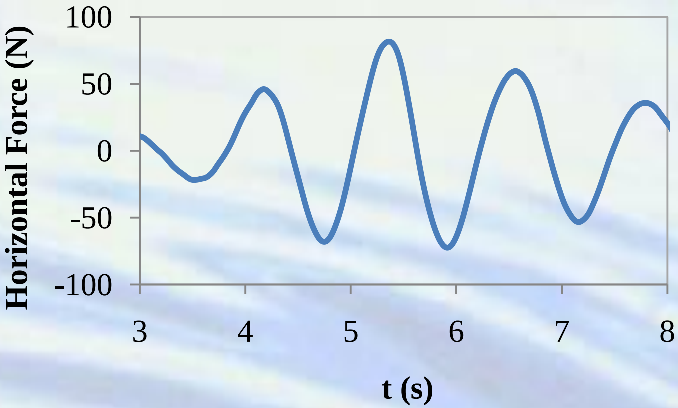
Comparison with experimental and numerical results for free surface elevation at focus point

4. Wave impact on structure

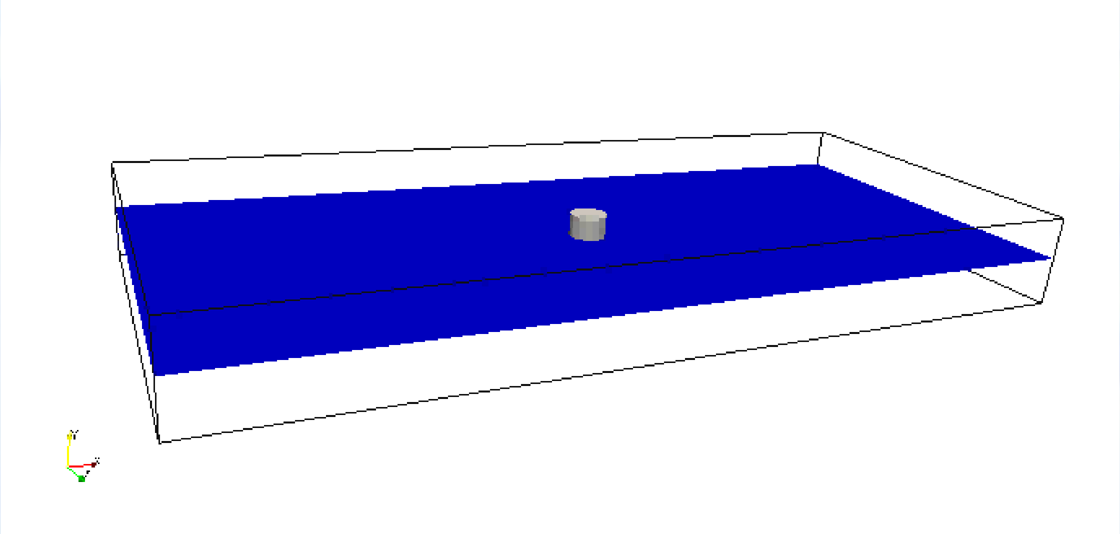
4.3 Extreme wave generating test



Input wave spectrum



Time history of horizontal force on cylinder



Extreme wave impact on cylinder

5 . *Summary*

- Scour is a threat to the stability of an offshore wind turbine structure.
- Wave loading on an offshore wind turbine mount is an important factor for its stability.
- **Numerical model has been developed to simulate the scour development at the base of monopile mounts and predict the depth of the scour hole.**
- **Different kinds of wave impact on a range of mount configurations can be simulated and the resulting loadings can be calculated.**
- Computational fluid dynamics (CFD) modelling at MMU will be combined with laboratory experiments at Lancaster in the future.

Thank you for your attention