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Numerical Modelling of Wave Interaction with Porous Structures

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Abstract This paper presents a numerical model for simulating wave interaction with porous structures. By using the free surface-capturing approach together with a novel Cartesian cut cell treatment, the Finite Volume Model calculates the two phase flows out side of porous structure based on the Navier-Stokes equations, while the flow in the porous structure is described by Navier-Stokes type model equations. The free surface of water is treated as a contact discontinuity in the density field which is captured automatically as part of the numerical solution by using a time-accurate artificial compressibility method and high resolution Godunov-type scheme. The numerical model is first calibrated by simple test for a steady flow passing through a porous block. Reasonably good agreements with other numerical results are obtained. After that, the numerical model is used to simulate the breaking wave overtopping a caisson breakwater, protected by a layer of armor units. The results show that the porous armor layer is effective in reducing the overtopping rate as well as in protecting the stability of the caisson breakwater.

Key words: two phase fluids, porous structure, numerical model, wave overtopping

INTRODUCTION

Porous armor layers are widely used to protect coastal structures, i.e., seawalls or caisson breakwaters, from wave attack. To determine the stability of the armor layer as well as the protected coastal structure, the knowledge of the flow motion in porous media and the corresponding pressure and force fields is needed. The computational study of a viscous free surface flow through porous structure is one of the most challenging topics, but the results, which may elucidate more detailed mechanisms, are both essential and desirable in both the research and engineering communities.

The surface capture technique has proven to be a feasible and accuracy method for computing the motion of incompressible fluids with free surface [1∼3]. It using a high resolution shock capturing solver to capture the discontinuous density profile which occurs at the free surface and in contrast to VOF schemes no explicit surface reconstruction is required. However, in order to capture the free surface the governing equations must be cast in a hyperbolic formulation requiring the use of a dual time approach to solver the incompressible form of the Navier-Stokes equations [4].

To model wave interaction with porous structures, Liu et al. [5] developed a numerical model in which the flow in porous media is described by Darcy spatial-averaged Navier-Stokes equations. Later, Huang et al. [6] took Navier -Stokes type model equations for the flow inside the porous media which were obtained by adding the convective inertial force term and viscous force term into the equations. Following these works, we extended the free surface capturing flow solver AMAZON-SC for modeling the porous media. After validating it with other numerical results, the model is then applied to simulations of breaking wave overtopping a caisson breakwater, protected by a layer of armor units.

NUMERICAL METHOD

For incompressible, unsteady, viscous flows, the two-dimensional Navier-Stokes equations with a variable density field can be modified by the artificial compressibility method and written as the integral form,

$$\frac{\partial}{\partial t}\iint_{\Omega}\boldsymbol{Q}\mathrm{d}\Omega + \oint_{S}\boldsymbol{F}\cdot\boldsymbol{n}\mathrm{d}s = \iint_{\Omega}\boldsymbol{B}\mathrm{d}\Omega \qquad (1)$$

where Ω is the domain of interest, S is the boundary surrounding Ω, $\boldsymbol{n}$ is the unit normal to S in the outward direction. $\boldsymbol{Q} = [\rho, \rho u, \rho v, p/\beta]^{\mathrm{T}}$ is the vector of conserved variables. $\boldsymbol{F}$ is the vector of flux function, including both inviscid and viscous fluxes, through S. $\boldsymbol{B}$ is the source term of body forces and β is the artificial compressibility coefficient.

To achieve a time accurate solution for these hyperbolic governing equations, an implicit dual time iteration technique has been used, in which the solution at each real time step is obtained by solving a steady state problem in a pseudo time domain. For evaluating the inviscid fluxes, Roe's flux function is adopted locally at each cell interface assuming a one dimensional Riemann problem in the direction normal to the cell face, while the viscous fluxes are discretized using central difference approximations directly at a given cell face. In the pseudo-time iteration, the resultant linear equations are solved using an approximate LU factorisation scheme [7]. At every real time step, once the flow variables including density have been calculated, the position of the free surface can be defined as the contour with the average density value of the two fluids. A novel scheme has also been proposed by Qian et al. [2], for the accurate treatment of the pressure gradient term within the free surface capturing method for flows under the influence of gravity, where the vertical pressure gradient term is split into hydrostatic and kinematic terms which are then calculated separately in order to exactly balance the gravity source term in each cell. For full details of the solution method the reader is referred to [2, 3], the following discussion will consider only the extension to porous structures.

In order to extend the solver to deal with porous structures, a porosity model must be included. To achieve this, the body force term of the Navier-Stokes equations are extended to include terms modelling the flow resistance of the porous medium using the method proposed by Huang et al. [6].

$$\boldsymbol{B} = \begin{pmatrix} 0 \\ -\dfrac{\upsilon N_w}{K_p}u - \dfrac{C_f N_w^2}{\sqrt{K_p}}u\sqrt{u^2+v^2} \\ -\dfrac{\upsilon N_w}{K_p}v - \dfrac{C_f N_w^2}{\sqrt{K_p}}v\sqrt{u^2+v^2} - \rho g \\ 0 \end{pmatrix} \qquad (2)$$

where υ is the kinematic viscosity, K_p (m^2) is the permeability coefficient, N_w is the dimensionless intrinsic porosity and C_f is a dimensionless turbulent resistance associated with the structure. In general N_w is a design parameter of the structure and is known along with a nominal diameter d_n of the porous structure. For computing K_p and C_f, the following formulations are used:

$$K_p = 1.643\times 10^{-7}\left(\frac{d_n}{d_0}\right)^{1.57}\frac{N_w^3}{(1-N_w)^2}\ , \qquad \text{where } d_0\text{=10 mm} \qquad (3)$$

$$C_f = 100\left(d_n\sqrt{\frac{N_w}{K_p}}\right)^{-1.5} \qquad (4)$$

To complete the numerical solution the intrinsic permeability and porosity coefficients are stored at each grid cell in the computational domain. The additional terms in the body force vector are then computed at each porous grid cell. Porous structures are thus represented by defining a region of grid cells with non-zero N_w.

MODEL APPLICATIONS

Flow passes a porous block

In order to test the implementation of the porosity model, the solver was first applied to the test case described by Fu et al. [8] and shown in Figure 1. A porous block with height 0.5H is located at

7.5H away from the left boundary of a horizontal channel with height H and length 58H, respectively. The velocity is prescribed at the left hand (inlet) boundary to ensure a fully developed parabolic distribution with average value u_0, i.e. $u = 6y(1 - y)$. In the present case u_0 is chosen to ensure an inlet Reynolds number of 500, the porosity of the porous block is 0.5 and the particle diameter is 0.05H (H = 1.0m is specified for the test). The simulation is run until a fully developed flow solution is obtained (Figure 2), the results compare favourably with those reported by Fu et al. [8] and give confidence in the porosity model.

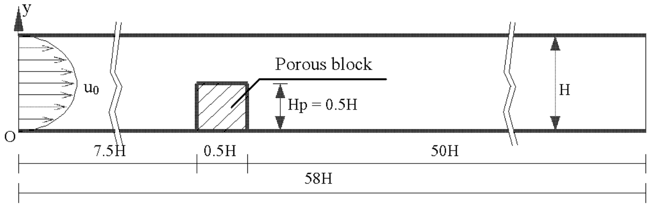


Figure 1: Schematic of flow in a channel with a porous block

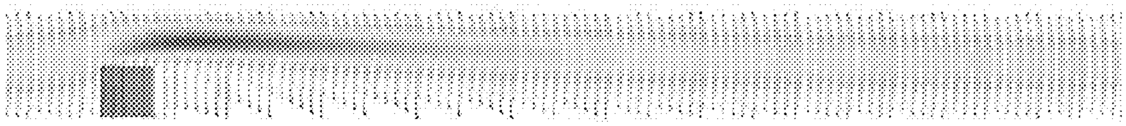


Figure 2: Velocity field for fully developed flow at around the porous block

Breaking waves overtopping caisson protected by porous armor units

We choose to simulate the same test case which Liu et al. [5] used. In that case, the caisson breakwater is impermeable, whereas the armor layer in front of the caisson is made of concrete units called tetrapod with a maximum height of 0.063m and porosity of 0.50. The effective diameter of the tetrapod is estimated as $d_{50} = 0.05$m. The wave parameters are following: wave period 1.4s, wave height 0.105m, and still water depth 0.28m.Three wave gauges are located at 0.27m, 3.87m and 7.02m away from the left boundary, respectively, to measure the time history of the free surface displacement. The computational domain is shown in Figure3.

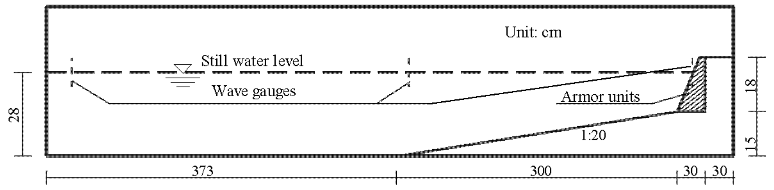


Figure 3: Computation domain for breaking waves overtopping Caisson with porous armors unit protected

The simulation time lasted 14s, which corresponds to a wave train of 10 waves. The velocity and density distributions on the left inflow boundary are specified according to the linear wave theory for the given parameters of wave trains. The calculated time histories of the free surface displacements at three sections are shown in Figure 4. Corresponding with the results of Liu et al. [5] which is not shown here, the overall agreement is very good. During the simulation period, there are 6 overtopping event occurred which can be seen clearly in Figure 5. The average overtopping rate is about 2.18 kg/m·s^{-1}, which is less then the laboratory experiments results (2.62 kg/m·s^{-1}) and the numerical result of Liu et al. (2.78 kg/m·s^{-1}). This maybe due to the interface was treated by using the average density of the two fluids. When the wave overtopping the Caisson and breaking, this processes cause the jet to have entrained a significant amount of air, which could leading the density of some detective areas less than the average value but still contain some water. The numerical result without porous

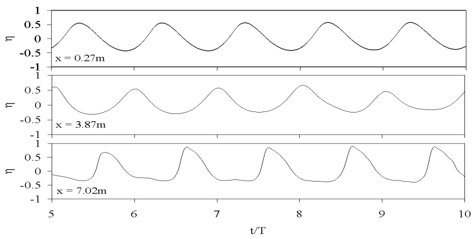


Figure 4: The time history of the free surface at three wave gauge points

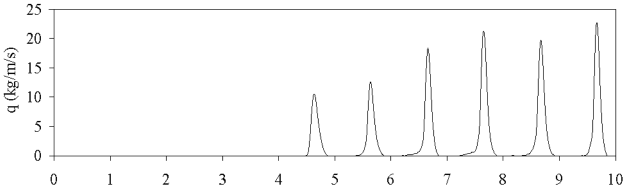
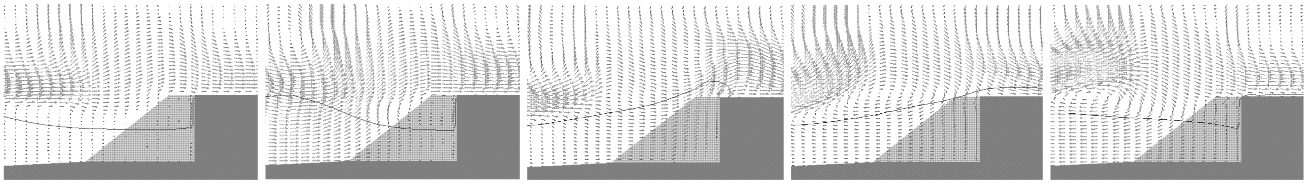


Figure 5: The time history of the instantaneous discharge at front of Caisson

armors units shows that the average overtopping rate is about 2.96 kg/m/s, which is much higher then the result with porous armors units.

Figure 6 shows the corresponding velocity field in front of the caisson breakwater, which demonstrates directly the protective role of the porous armor layer. It can be seen that the porous layer acts as a buffer absorbing the wave energy and attenuates the flow impact on the caisson. The velocity decreases toward the core of armor units and nearly approaches zero near the toe of the caisson. Hence, the scour of the caisson by the continuous wave action is greatly reduced due to the use of the armor units.



(a) 7.2 (b) 7.4 (c) 7.6 (d) 7.8 (e) 8.0

Figure 6: Simulated velocity filed in front of Caisson with Porous Armor Layer at t/T

CONCLUSIONS

A surface-capturing FV model is enhanced for solving the flow in porous media based on Navier-Stokes type equations. The practical application indicates that the model is capable to simulate wave interaction with complex porous and solid structures. The results show that the porous armor layer is effective in reducing the overtopping rate as well as in protecting the stability of the caisson breakwater.

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