SIZE RELATED PERFORMANCE LIMITATIONS ON WIND TURBINE CONTROL PERFORMANCE

S. Dominguez * **W. E. Leithead** *

* *Industrial Control Centre*
University of Strathclyde
Glasgow. U.K.

Abstract: Wind turbine size has increased exponentially for the last twenty years. With the increase in size the control systems for variable speed pitch regulated machines play a greater role, since they are used for other tasks than just regulating generator speed. The structural dynamics impact directly on the achievable control system performance becoming more restrictive as the size of wind turbine increases. To make this dependence explicit, the relationships between performance measures of the control system and wind turbine structural dynamic characteristics are derived.

Keywords: Fatigue loads, feedback, control, pitch, fundamental limitations, right half-plane zeros, time delay, bandwidth

1. INTRODUCTION

Over the last twenty years, the size of wind turbines has increased rapidly, see Figure 1. The interest in continuing this trend is obvious, but whether there is a limit to the possible size of wind turbines, assuming the current dominant design, is unclear. Furthermore, as wind turbines get bigger, interest in lighter materials and more flexible tower and blades increases. The issue arises as to whether some combination of these factors, bigger wind turbines and more flexible structures, might lead to a limitation on the size from a part of the wind turbine design which is typically not considered critical: the control system.

In recent years there has been renewed interest in studying the limitations imposed on control systems by the structural characteristics of the plant to be controlled. It is well known that when a plant is non-minimum phase the benefits of feedback are reduced (Horowitz, 1963). The non-minimum phase effect is caused either by right-half plane zeros, time delays or sampling, characteristics all present in the wind turbine control problem. The main effect of the non-minimum phase nature of the plant is a limitation on the achievable bandwidth (Sidi, 1997) and constraints on the sensitivity and complementary sensitivity functions (Freudenberg and Looze, 1987).

Sidi has studied the limitations on the bandwidth imposed by different non-minimum phase elements: in (Sidi, 1980), the limitations imposed by the non-minimum phase nature of sampling is investigated; in (Sidi, 1997) criteria to estimate the limits on the bandwidth imposed by right half plane zeros (RHPZs) and poles are given; in (Sidi and Yaniv, 1999), previous results from (Sidi, 1997) are extended to the case with several right half-plane poles and zeros. In the aforementioned references only the case with single RHPZs and poles present is examined.

In this paper the limitations derived in (Sidi, 1997) are extended to cover the limitations imposed by a pair of complex RHPZs. These limitations are applied to wind turbine control, where they are especially relevant, given the non-minimum phase nature of the wind turbine.

2. MODELS AND DYNAMICS

The linear model for the wind turbine dynamics used in this paper is that reported in (Leithead and Rogers,

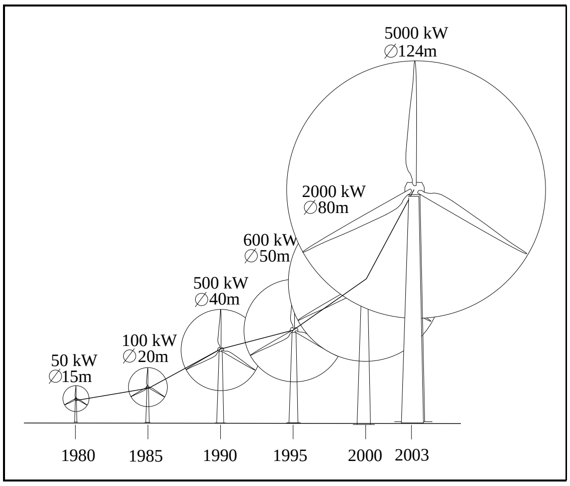

Fig. 1. Wind turbine size evolution

1996; Leithead and Connor, 2000). The model has been validated against a variety of real wind turbine data and FLEX simulations, see Figure 2 where the directly identified response of a detailed FLEX simulation is compared with the frequency response of the linear model for a multi-Megawatt wind turbine. This linear model is being integrated into a MATLAB Toolbox for wind turbine controller design.

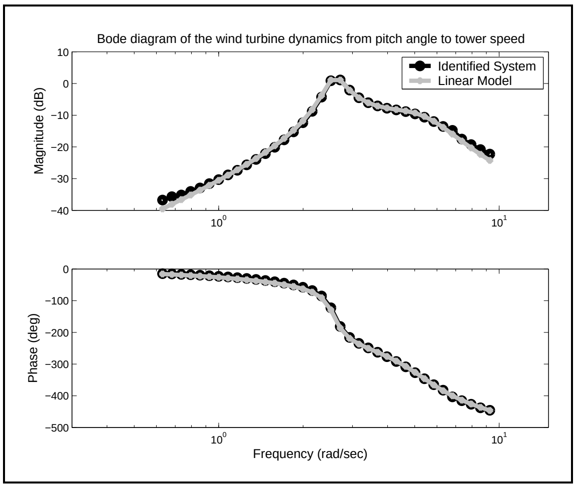

Fig. 2. Identified frequency response and linear model frequency response

The linear model includes all the dynamic components significant for controller design and control performance assessment, in particular, two modes for the tower, two modes for the blades and two modes for the drive-train. It also includes the dynamics of the pitch actuator and the interaction of the rotor with the wind. The main differences to other linear models of wind turbine dynamics found in the control literature is the explicit inclusion of the tower and blade modes. The tower modes are of particular importance in the design of controllers for wind turbines since they introduce a pair of RHPZs which impose limitations on the generator speed loop (Seron *et al.*, 1997; Sidi, 1997). The blade modes are important since the blade flap mode interacts with the tower mode and the blade edge mode is a major contribution to the first drive-train mode.

In Figure 3, the Bode plot for the transfer function representing the dynamics from pitch demand to generator speed for a commercial multi-Megawatt wind turbine is depicted for two different wind speeds. At $13m/s$, the presence of lightly damped RHPZs arising from the tower dynamics at about $2rad/s$ is evident. The acute phase loss due to these RHPZs influences strongly the maximum bandwidth that the generator speed controller can achieve. At $16m/s$, the RHPZs due to the tower are no longer present in the plant dynamics, since they move into the left-half plane as the blade pitches. The pitching of the blades, and the change in the relationship between flap and edge mode, are also responsible for the changes in the Bode magnitude plot at mid frequencies in Figure 3. In Figure 4 the location as wind speed varies of the

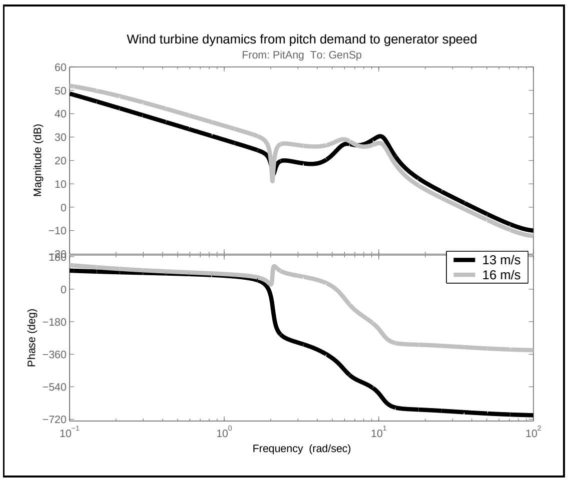

Fig. 3. Dynamics from pitch angle to generator speed

RHPZs due to the tower dynamics is depicted. The physical explanation of these right-half plane zeros is the following: given an increase in pitch angle in order to reduce the aerodynamic torque, the thrust is reduced and the rotor displacement adjusts by moving forward. During this motion the wind speed relative to the rotor is increased, causing the aerodynamic torque also to be transiently increased. Hence a control action aimed at reducing the torque causes an initial increase.

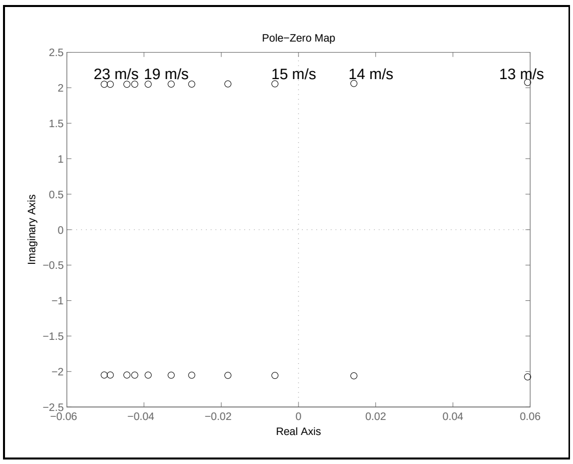

Fig. 4. Right half plane zeros associated with the tower

In Figure 3, it is seen that there is an additional pair of RHPZs. The second pair arises from the dynamics relating rotor torque to hub torque, T_β, which depend on the pitch angle, the blade flap and edge-wise modes and the aerodynamic gains. The derivation of T_β is explained in (Leithead and Rogers, 1996), but a simplified expression from which the RHPZs associated to these dynamics can be found is

$$T_\beta = T_{\beta 1} + T_{\beta 2} \tag{1}$$

with

$$T_{\beta 1} = \frac{\left[s^2 \left(F_{1\beta} - (\omega_e^2 - \omega_f^2) sin(\alpha) cos(\alpha) F_{1\beta}/\omega_e^2 \right) + \atop + s(F_{1\beta} L \frac{\partial F2}{\partial w}/J - F_{2\beta} L \frac{\partial F2}{\partial w}/J) + \omega_f^2 F_{1\beta} \right]}{(s^2 + s(L\frac{\partial F2}{\partial w}/J) + \omega_f^2)(s^2 + a_2 s + b_2)}$$

$$T_{\beta 2} = \frac{(s^2 + \omega_e^2) \left[\beta_2 \left(1 - \frac{F_{1\beta}}{\partial F_1 / \partial \alpha} \right) \right]}{\omega_e^2 (s^2 + \eta s + \mu)}$$

where

$$F_{1\beta} = \frac{\partial F_1}{\partial \alpha} + \frac{(\omega_e^2 - \omega_f^2)\omega_e^2 w_0 \frac{\partial F2}{\partial w}}{2\omega_e^2 \omega_f^2}$$

$$F_{2\beta} = \frac{\partial F_2}{\partial \alpha} + \frac{(\omega_e^2 - \omega_f^2)\omega_f^2 \left((w_0 \frac{\partial F1}{\partial w} + \Omega_0 \frac{\partial F1}{\partial \Omega_0} \right)}{2\omega_e^2 \omega_f^2}$$

$$\mu \approx \frac{K_R \bar{K}_{Ls}}{J_R (K_R + \bar{K}_{Ls})} \Big/ \left[1 + \frac{I_{Ls}}{J_R} \frac{K_R^2}{(K_R + K_{Hs})^2} \right]$$

$$\eta \approx \frac{B_R}{J_R} \frac{I_{Ls}}{J_R} \frac{K_R^2}{(K_R + \bar{K}_{Ls})^2} \Big/ \left[1 + \frac{I_{Ls}}{J_R} \frac{K_R^2}{(K_R + K_{Hs})^2} \right]$$

and

$$\omega_e^2 = \frac{K_E + J\dot{\theta}_{R_0}^2}{J}$$

$$\omega_f^2 = \frac{K_F + J\dot{\theta}_{R_0}^2}{J}$$

The notation used in this paper is the same as in (Leithead and Rogers, 1996). The location of the zeros associated with T_β is depicted in Figure 5.

Although (1) is a simplification of the dynamics from aerodynamic torque to pitch angle, it is nevertheless a good approximation. It is not only useful for locating the RHPZs, but also for investigating the relationship of the location of the zeros to the different structural parameters. The frequency of the RHPZs associated with T_β depend on the structural characteristics of the wind turbine. As can be seen from (1), these zeros are mainly dependent on the blade flap and edge frequencies and on the aerodynamic gains. The dependence of the RHZPs of T_β on the blade edge mode frequency is negligible and there is no dependence whatsoever on tower frequency. However, these RHPZs have a very strong dependence on the blade flap mode frequency, see Figure 6 where the location of the zeros as flap frequency varies from $0.6Hz$ to $1.5Hz$ is depicted for various wind speeds.

As can be seen from Figure 4, the damping of the zeros due to the tower, which are almost exactly at tower frequency, is very small. These zeros induce a sudden drop in the phase at tower frequency, imposing a very simple constraint: the cross-over frequency of the controller can never be greater. However, the zeros associated with T_β, are heavily damped and their frequency varies. This enhances the possibility that the phase loss they induce spreads towards low frequencies, limiting the controller performance.

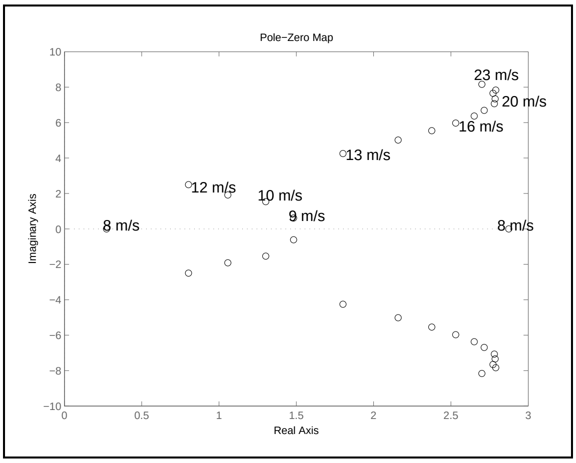

Fig. 5. RHPZs associated with T_β

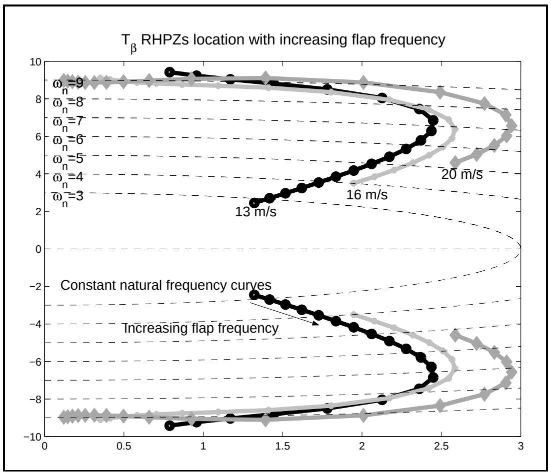

Fig. 6. Variation of the frequency of zeros with wind speed for the 5MW machine

3. LIMITATIONS IN BANDWIDTH IMPOSED BY RIGHT HALF-PLANE ZEROS AND TIME DELAYS

In Figure 7, a typical example of the Bode plot for open loop dynamics, $L(s)$, is depicted. In the vicinity of ω_c, the controller cross-over frequency, $L(s)$ can be approximated by

$$L^*(s) = \left(\frac{a}{s} \right)^k \tag{2}$$

Bode's ideal loop transfer function. For (2), the amplitude curve has a constant slope of $-k$ and the phase curve is a constant line at $-k\pi/2$. This transfer function is supposed to approximate the open loop transfer function in the vicinity of ω_c, the open loop cross-over frequency, and should remain valid until more or less ω_p, the phase cross-over frequency. Assuming this approximation to be valid, it is straight forward to analyse the loss of phase induced by a pair of RHPZs, and its subsequent impact in the open-loop crossover frequency. The development follows (Sidi, 1997; Seron *et al.*, 1997).

Assuming that $L(s)$ is non-minimum phase, it can be split into

$$L(s) = L_{MP}(s)L_{NMP}(s) \tag{3}$$

where L_{MP} is the minimum-phase component and L_{NMP} is the non-minimum phase component. The non-minimum phase component can be expressed as a Blaschke product, which is an all-pass function,

$$L_{NMP}(s) = \frac{(s^2 - 2\zeta\omega_z s + \omega_z^2)}{(s^2 + 2\zeta\omega_z s + \omega_z^2)} \tag{4}$$

With $0 < \zeta < 1$. It is obvious that this transfer function has a phase lag at frequency, ω, of

$$arg(L_{NMP}(j\omega)) = -2tan^{-1}\left(\frac{2\zeta\frac{\omega}{\omega_z}}{1-\left(\frac{\omega}{\omega_z}\right)^2}\right) \tag{5}$$

Since the minimum phase of the open loop transfer function can be approximated by Bode's ideal loop transfer function, for a certain k, which has a constant phase lag of $-k\pi/2$, the overall phase lag of $L(s)$ at a frequency ω is

$$arg(L(j\omega)) = -k\frac{\pi}{2} - 2tan^{-1}\left(\frac{2\zeta\frac{\omega}{\omega_z}}{1-\left(\frac{\omega}{\omega_z}\right)^2}\right) \tag{6}$$

Therefore, at the open loop cross over frequency, the phase lag in terms of the PM is,

$$-k\pi/2 - 2tan^{-1}\left(\frac{2\zeta\frac{\omega_c}{\omega_z}}{1-\left(\frac{\omega_c}{\omega_z}\right)^2}\right) = -\pi + PM \tag{7}$$

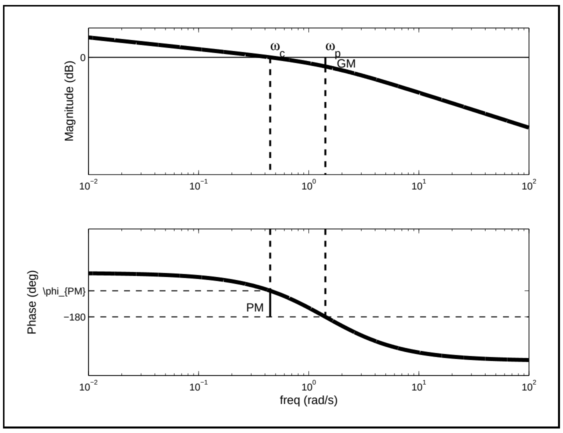

Fig. 7. Gain and phase stability margins

An expression relating the ratio between the controller crossover frequency and the frequency of the pair of zeros, ω_z, is, thus,

$$\left(\frac{\omega_c}{\omega_z}\right)^2 + \frac{2\zeta}{tan(\vartheta)}\left(\frac{\omega_c}{\omega_z}\right) - 1 = 0 \tag{8}$$

with

$$\vartheta = \frac{\pi(1-k/2) - PM}{2} \tag{9}$$

Of course, only the positive solution of (8) makes physical sense. Let this solution be $\Xi_r(k, PM)$. A similar relationship can be established for ω_p, namely

$$\left(\frac{\omega_p}{\omega_z}\right)^2 + \frac{2\zeta}{tan(\vartheta_p)}\left(\frac{\omega_p}{\omega_z}\right) - 1 = 0 \tag{10}$$

with

$$\vartheta_p = \frac{\pi(1-k/2)}{2} \tag{11}$$

the solution to which is $\Psi_p(k, PM)$. Recalling the definition of Bode's ideal transfer function,

$$20log\left[\left(\frac{\omega_p}{\omega_c}\right)^k\right] = GM \tag{12}$$

and, therefore,

$$\left(\frac{\omega_p}{\omega_c}\right)^k = \left(\frac{\Psi_p(k, PM)}{\Xi_r(k, PM)}\right)^k = 10^{(GM/20)} \tag{13}$$

Expression (13) is solved to find k for a given GM and a given PM. Having calculated k, the maximum crossover frequency possible for a system with a complex pair of RHPZs is obtained using

$$\omega_c = \omega_z \Xi(k, PM) \tag{14}$$

In the above, only one pair of complex RHPZs is assumed. Results obtained by the above procedure should be regarded as conservative. Limitations on the crossover frequency are given by all non-minimum phase elements in the control loop, including sampling and time-delays, both of which are present in the wind turbine control problem. The phase loss induced by these elements is straightforward to account for.

However, the limitations imposed by the RHPZs are not limited to the bandwidth of the controller. Lower bounds on the sensitivity function and the complementary sensitivity function are also imposed due to the presence of non-minimum phase elements in the transmission loop, see Figure 8, and these limitations play a fundamental role in the design process (Freduenberg *et al.*, 2000; Bode, 1945).

Due to the rotational nature of the wind turbine, deterministic peaks are present in the disturbance spectra at frequencies multiples of the rotor rotational speed. The high gain regions of the sensitivity function should not coincide with these deterministic peaks. The increase in the peaks of the sensitivity function also has implications in the control of the tower fore-aft movement using pitch control since, when trying to achieve several control objectives using the same control action with a multi-loop structure, the sensitivity function can give an estimation of the interaction between the various loops. Therefore, when the sensitivity function has a large positive value, the action of one loop is likely to interact with the other, which might lead to instabilities, see (Leithead and Dominguez, 2004).

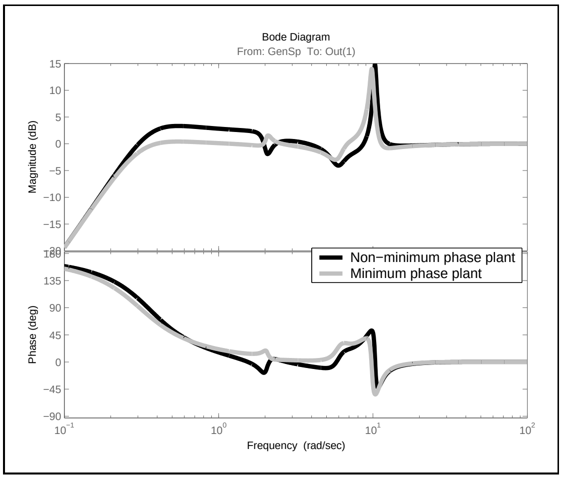

Fig. 8. Sensitivity function of the non-minumum phase plant and its minimum phase equivalent

4. EXAMPLE OF APPLICATION

The previous procedure is applied to two different wind turbines. Both machines share the same characteristics, up-wind, variable speed, three bladed multi-Megawatt wind turbines, with variable pitch capability for active control above rated. Both machines differ in generated power and size, one being a $3MW$ machine and the other a $5MW$ machine. The tower mode being at $2.64rad/s$ for the smaller wind turbine iand at $2rad/s$ for the bigger machine. In both cases a controller bandwidth of $1rad/s$ is required. In Figures 9 and 10, Figures 4 and 5 are repeated for the bigger machine.

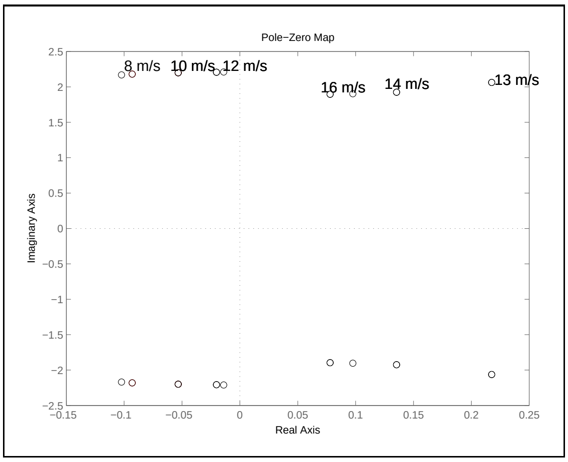

Fig. 9. RHPZs tower associated for the 5MW machine

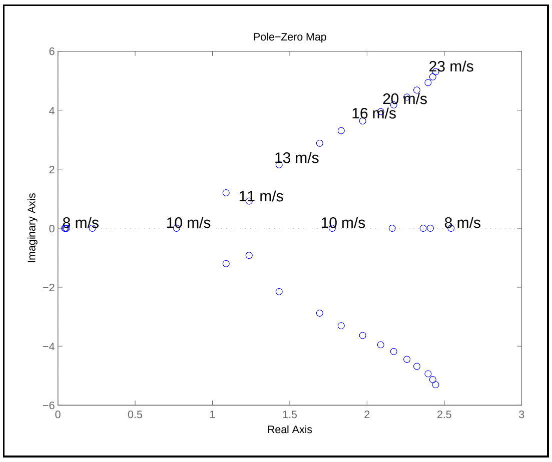

Fig. 10. RHZPs associated with T_β for the 5MW machine

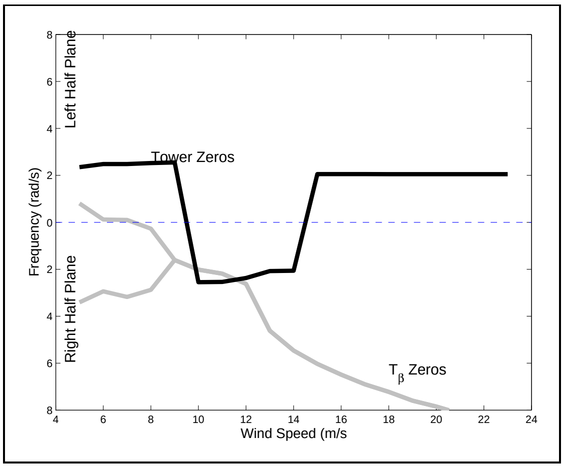

Fig. 11. Variation of the frequency of zeros with wind speed

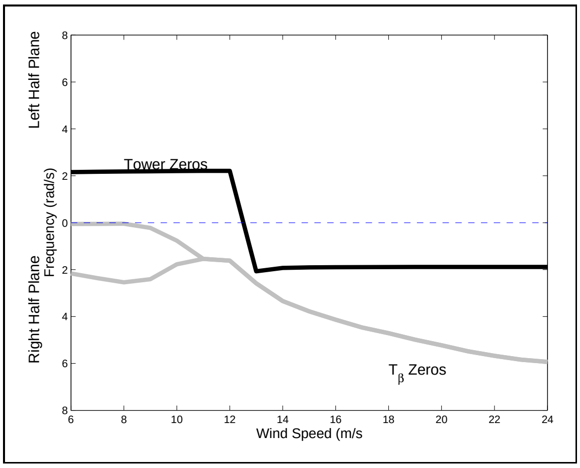

Fig. 12. Variation of the frequency of zeros with wind speed for the 5MW machine

In Figures 11 and 12 the frequency variation with wind speed of both the tower zeros and the zeros associated with T_β are shown, for both wind turbines. As mentioned, the frequency of the zeros associated with the tower remains constant, whereas the frequency of the zeros associated with T_β varies over a big range for both wind turbines, in both cases following a similar pattern. The zeros of T_β increase in frequency as wind speed raises, therefore becoming less critical for the control action.

The maximum bandwidth achievable with each one of these configurations is examined. In both cases, the wind turbines' rated wind speed is between 12 and $13m/s$. Furthermore, it is at these low wind speeds that the control requirements are most stringent, since the gain of the plant is lowest due to the low value of the gradient of the aerodynamic torque with respect to pitch angle. Therefore, it is at these low wind speeds, just above rated, that the limiting effect of the RHPZs is analysed.

At 12 and $13m/s$, the Blaschke products for the RHPZs associated with T_β has the following form for the smaller wind turbine

$$T_{\beta 12} = \frac{s^2 - 1.605s + 6.875}{s^2 + 1.605s + 6.875}$$
$$T_{\beta 13} = \frac{s^2 - 3.604s + 21.33}{s^2 + 3.604s + 21.33}$$

and for the bigger machine

$$T_{\beta 12} = \frac{s^2 - 2.176s + 2.624}{s^2 + 2.176s + 2.624}$$
$$T_{\beta 13} = \frac{s^2 - 2.474s + 6.673}{s^2 + 2.474s + 6.673}$$

Choosing safe stability margins, $10dB$ of GM, and 60^o of PM, at $12m/s$, the maximum achievable bandwidth for the smaller wind turbine is $0.65rad/s$ and, for the bigger wind turbine, is $0.27rad/s$. The latter reflects the effect of the lower frequency of the zeros. As the wind speed increases, hence increasing the frequency of the RHPZs, the maximum avalaible bandwidths increase to $1.04rad/s$, and $0.49rad/s$, respectively. This implies that with the chosen stability margins, a bandwidth of $1rad/s$ is unachievable at $12m/s$. For stability margins of $6dB$ and 45^o the maximum bandwidth is $1rad/s$ for the smaller machine although just $0.43rad/s$ for the bigger machine. In the latter case, the required bandwidth of $1rad/s$ cannot be achieved.

5. DISCUSSION AND CONCLUSIONS

In this paper, a procedure is presented that, with the input of readily avalaible parameters of the wind turbine, enables an estimate of the maximum crossover frequency possible for the pitch control system to be obtained. Traditionally, these issues had been given little or no importance, but with the current increase in size in wind turbines, the possibility of having a machine for which the pitch control system cannot reach the required bandwidth is realistic. Both the dynamics of the tower and the dynamics from aerodynamic torque to hub torque are relevant, witch each inducing a pair of complex conjugates RHPZs. The frequency of the pair associated with the tower varies little with wind speed remaining very close to tower frequency. The frequency and the damping of the pair associated with T_β changes as wind speed changes and is completely independent of tower frequency and blade edge mode frequency but strongly dependent on blade flap mode frequency.

The approach provides useful insight into the consequences of the choice of natural frequencies for blades and towers during the design process. It should be emphasized that the limitations raised with this method are intrinsic to the system, since they are derived from structural characteristics. This implies that the available bandwidth is a constraint independent from the control methodology chosen to design the controller. Therefore, if the avalaible bandwidth has a lower value than the intended bandwidth, the latter will be, by no means, achievable, resulting in a poor performance of the controller.

6. REFERENCES

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