Oldenburg University Workshop,
30th June - 2nd July, 2008

Aerodynamic Rotor Model for Unsteady Flow and Wake Impact

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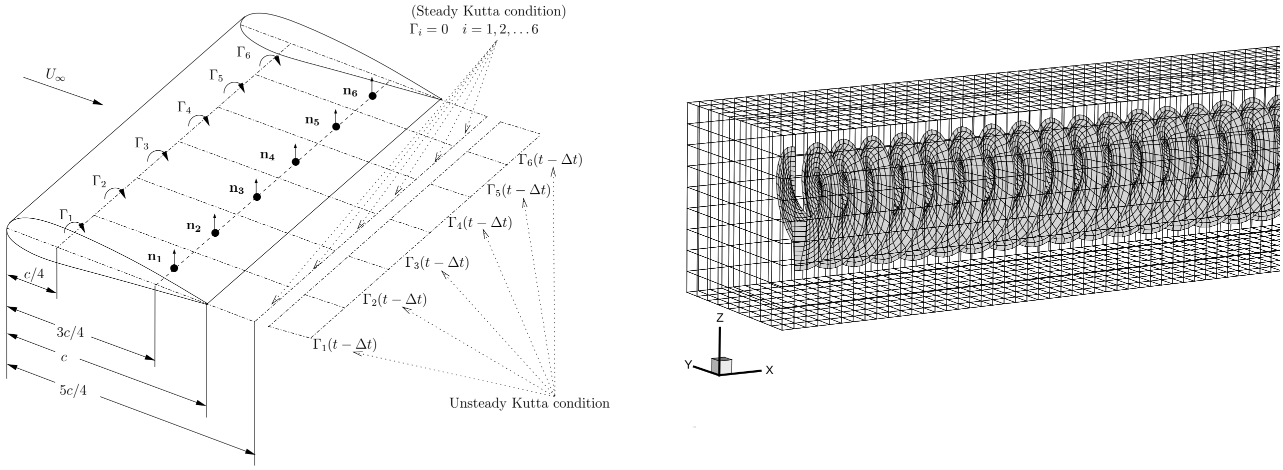
June 2008



(a) Illustration of the LLM method.

(b) Cartesian grid used for efficient computation of the wake induced velocity.

Figure 1: Computational method used for the present study.

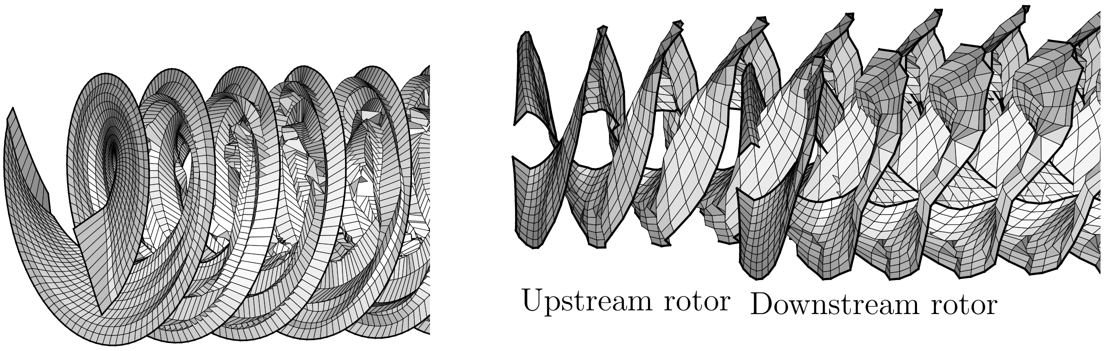
1 Abstract

A code has been developed based on free wake vortex lattice theory. The method includes viscous corrections to the blade section lift and drag, to model the aerodynamic response of a horizontal axis wind turbine rotor in unsteady flow. An indicial method is used to carry out sufficiently long computations to build up converged statistics of the response characteristics of the rotor in a turbulent flow. This model is being run currently in an incident flow modelling homogeneous turbulence. Following this, the model will be used to establish the statistics of unsteady forces induced by interaction of the rotor with incident wakes of upwind turbines in array configurations.

2 The Lifting Line Method

In the present study, a boundary integral (panel) method is used to calculate the inviscid flow around a wind turbine rotor. The blade is represented by a distribution of vortex line singularities according to the Lifting Line (or Weissinger's) method (LLM), see e.g. Katz, Plotkin (1991). This is essentially, a zero order approximation, for the inviscid flow around an airfoil, in terms of its thickness distribution (see figure 1(a)). The blade circulation is concentrated at the blade quarter chord and the wake sheds at one quarter chord behind the trailing edge. Thus, the wake is a succession of discrete vortex rings and the wake panel elements convect according to the local flow velocity.

To model viscous effects, a viscous correction is applied. Tabulated data for the airfoil's 2D static lift coefficient are used to modify accordingly, the strength of the last shed vortex rings. Static 2D drag coefficient data are also used to model the profile drag effect on the blade aerodynamic forces. The method allows a free wake representation, applying directly Biot-Savart law for the calculation of the wake induced velocity components. As the number of wake panels in the flow field increases, this process for the free wake evolution becomes



(a) Free wake computation for a single rotor in a uniform flow.
(b) Staggered rotor wake interaction immersed in an ABL model.

Figure 2: Free wake computations for a wind turbine rotor.

progressively more inefficient. In order to decrease the computational work required, a linear interpolation technique is employed. A Cartesian grid is created at the beginning of the computation, covering the rotor and extending a sufficient region downstream and upstream of the rotor plane, see figure 1(b). The velocity at the grid's nodes is calculated by the Biot-Savart law. The vortex filaments convect according to the local velocity of their end points. The local velocity is computed by linear interpolation from the grid.

A free wake computation (using the Biot Savart law directly) for a single rotor operating in a uniform stream, is shown in figure 2(a). A free wake computation using the wake grid method is shown in figure 2(b), for the case of two rotors in a tandem configuration, operating in a logarithmic incident mean velocity profile, modelling the mean velocity component of an idealised atmospheric boundary layer (ABL).

In order to validate the present LLM method, results are compared with those obtained by the Blade Element Momentum (BEM) method, including the tip and root correction according to Prandtl, see e.g. Glauert (1943). The comparison is done for a preliminary rotor model, for which experimental measurements were performed in the $3m \times 1.5m$ Honda Wind Tunnel, at Imperial College, London. The comparison is based on the turbine's mean power coefficient, $\overline{C_P}$, in terms of the tip speed ratio Λ. The resulting $\overline{C_P}(\Lambda)$ curves are shown in figure 3 for all three methods.

3 The Indicial Response Method

One of the goals of the present study is to investigate the turbine's response to wind turbulence effects, such as due to ABL turbulence and/or due to wake turbulence. For such an investigation a large amount of data is required to obtain converged statistics. The computational cost for this, using the current LLM code is high. Alternatively, an indicial method, using Duhammel's integral is applied. This is an exact method for linear systems, but it has been shown by Pesmajoglou, Graham (2000), to be a good approximation for

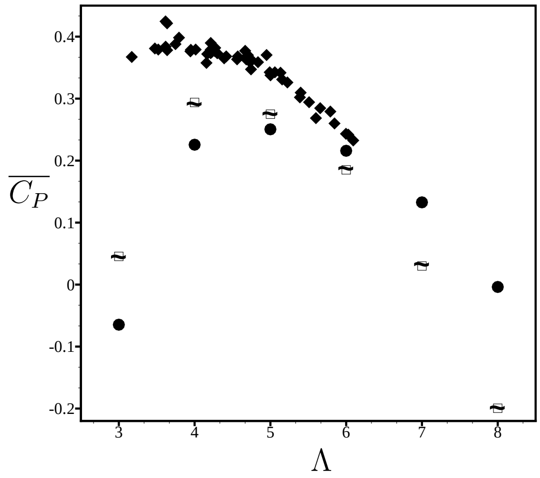


Figure 3: $\overline{C_P}$ versus Λ curves, by LLM and BEM compared with the wind tunnel experimental measurements. ◆, Wind tunnel measurements; ●,BEM; ∼, LLM(wake grid); □, LLM(Biot-Savart).

the non-linear rotor system. The basic steps of the method are as follows: a) obtain the turbine's response (whatever characteristic is required, e.g. axial force) to a unit impulse b) create a simulated time history for the turbulent velocity components (the forcing function) c) use Duhammel's integral to obtain the rotor's response to the turbulent forcing.

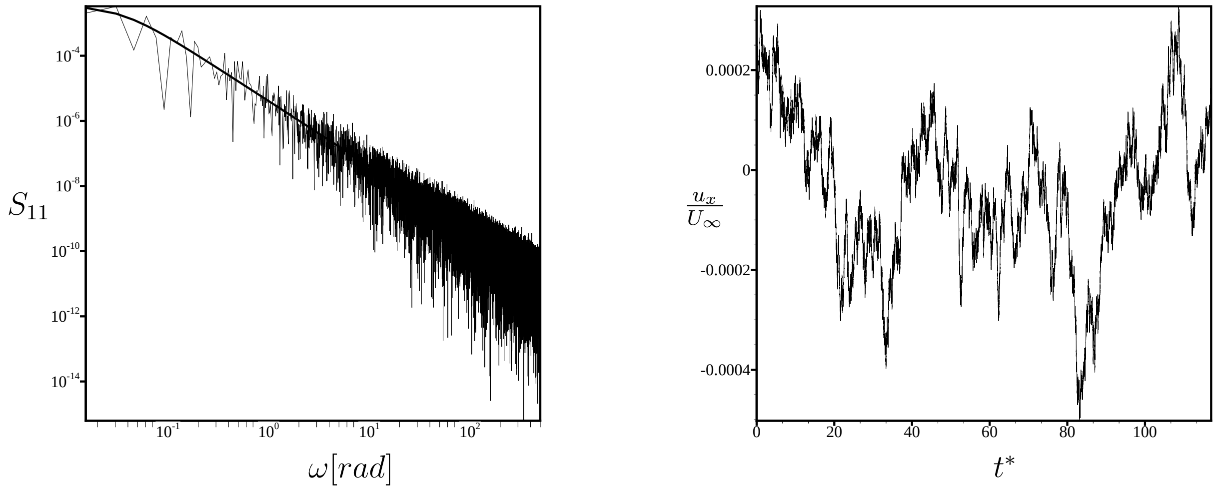
Currently, the rotor's response to the longitudinal velocity component is being considered. A random one-dimensional turbulent velocity field is simulated to fit the von Karman spectrum, which gives a power spectrum for the longitudinal component S_{11}:

$$S_{11} = \frac{\kappa^2}{[1 + (\Xi\omega)^2]^{5/6}}$$

where $\kappa = \sqrt{2\sigma^2 L_1/\pi}$, $\Xi = 4\pi^2\Gamma(1/3)L_1/(\sqrt{\pi}U_\infty{}^2\Gamma(5/6))$, ω is the radian frequency, σ the root mean square of the time series of the velocity component, L_1 the integral length scale of the turbulence, U_∞ the mean longitudinal velocity component assumed uniform and Γ denotes the Gamma function. The S_{11} spectrum and the modelled time series for the longitudinal velocity component are shown in figures 4(a) and 4(b), respectively. t^* denotes a dimensionless time defined as $t^* = U_\infty t/D$, where t is the time and D the rotor's diameter. The power coefficient C_P and the thrust coefficient C_T response of the rotor system to the forcing of figure 4(b) is shown in figure 5(a) and 5(b).

4 Conclusions and Future Work

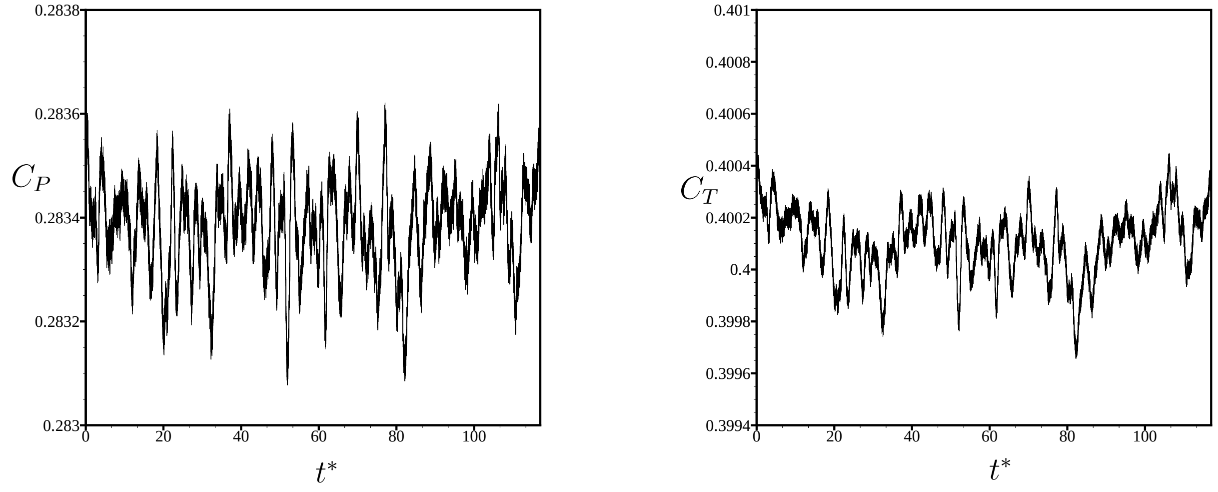
A Lifting Line method, including a viscous correction has been presented. The method will be used as a base method for studying wake effects on wind turbine rotor aerodynamics. For computational efficiency purposes, the LLM model will be used in conjunction with an indicial method, to obtain efficiently converged statistics. The computational results will be complemented with wind tunnel experimental measurements of the aerodynamic characteristics of wind turbine models, operating in upstream wind turbine's wakes.



(a) von Karman spectrum for the longitudinal velocity component.

(b) Modelled time series for the longitudinal component.

Figure 4: Modelling the longitudinal velocity component.



(a) Power coefficient response.

(b) Thrust coefficient response.

Figure 5: C_P and C_T rotor response subject to the forcing of figure 4(b).

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